ACME Advisory Committee on Mathematics Education

Part B - ACME's response to the draft Primary National Curriculum for Mathematics published on 11 June 2012

This part of the ACME response aims to identify detailed commentary on the statements included in the curriculum, with specific suggestions for changes, including correction of mathematical errors, highlighting mismatches in progression between and within strands, noting potential improvements in phraseology and suggestions for movement or deletion of various statements. The two parts must be viewed and acted on together – it is not sufficient to implement only the specifics highlighted in the second part, as the overarching issues also require focused attention by knowledgeable people.

This compendium of comments should not be regarded as complete, rather it points to the necessity for a small group of people to work together to ensure mathematical accuracy and coherence as part of the final process for the next draft.

Detailed comments on the aims are made in Appendix A and on content within the curriculum are made in Appendix B. Below we have identified areas that are working well, and those which are lacking or need further work.

Recommendation 30: mathematical experiences

Several respondents pointed to the need for all children to have a statutory experience of certain common mathematical representations, such as the number line, combination grids, and graphical representations, as well as informal representations before formal written formats. Use of strong images and visualisation is not a matter of pedagogy, because being able to call on these should be an objective in itself. In particular, the image of the number line allows connections to be drawn between strands (e.g. between number and measure, and graphing), which adds essential coherence to mathematical learning.

ACME recommends that there should be sufficient information in the curriculum to ensure core developmental experiences, such as use of the number line, which ensure progress towards conceptual understanding and fluency.

Recommendation 31: number and place value – size of numbers

We welcome the emphasis in the draft on learning number facts. We also support the learning of number bonds and multiplication facts as an aid to the stated aim of fluency.

There is a sense in which the draft assumes that working with numbers becomes increasingly more demanding as the size of the numbers grows. For instance in the sections on number, place value, approximation and estimation, pupils are expected to read, write and compare numbers to 100 in Year 3, followed by 1,000 in Year 3, 10,000 in Year 4, 1,000,000 in year 5 and then 10 million in Year 6. Calculating with 4 digits is no more conceptually challenging than with 2 digits – only the scope for error is increased with the growing number of sub-calculations. The numbers 11-20 are the only "irregular" sequence and by limiting children to these numbers we hide the simpler structure of the rest of the system from them.

There is scope for removing the increasing size of the calculation, and larger numbers should be introduced earlier. This would create space for statements that raise aspirations for conceptual understanding further.

Recommendation 32: number and place value – the importance of 10

During the workshops, several participants talked about making sure the importance of '10' is threaded through the curriculum and that understanding this should be an aim in itself.

Recommendation 33: measure

It is critically important that the play-based EYFS curriculum, which where appropriate uses informal units and non-standard recording, is extended and built upon in KS1 to ensure depth of conceptual understanding. For instance, the process of counting needs initially to be tied to the idea of counting 'things' before the abstract idea of a sequence of numbers is developed – otherwise counting becomes just reciting a series of words without a clear foundation in the notion of enumeration of objects and quantities.

Whilst we recognize the presence of a progression in the measurement strand there are nevertheless many serious inconsistencies which fail to match important developments in decimals and fractions, and also fail to develop an understanding of the quantities being measured. The coordination of measure and decimals needs to be developed with great care.

The development of measure should be redrawn to relate more closely to fractions and decimals and to the quantities being measured. Account should be taken of extensive research in this area.

Recommendation 34: developing pre-algebra

We had asked specifically about algebra in the light of the findings of for the ICCAMS project at King's, but algebra was also mentioned in response to other questions. Many respondents said that the PoS does not provide a coherent progression towards formal algebra, one claiming that it provided a weaker foundation than the current curriculum because the procedural approach discouraged thinking before acting. Many respondents suggested ways in which preparation for algebra could start in Year 1, and that expectations of algebraic thinking could be even more challenging if they were based on reasoning about relations between quantities, such as patterns, structure, equivalence, commutativity, distributivity, and associativity, and models and representations of these.

There are opportunities to be explicit about this throughout the current draft in relation to mental calculation methods and understanding direct and inverse relations. The early introduction of formal algebra before secondary school was not favoured by those who commented, as it is known to lead to poor understanding without a good foundation.

Algebra in primary connects what is known about number relations in arithmetic to general expression of those relations, including unknown quantities and variables. Operations need to be thoroughly understood in order to make this connection. The only generalising currently explicit in the PoS relates to pattern sequences and not other forms of relations.

A coherent developmental strand for algebra should be made explicit, making clear the connections between knowledge of number, mental methods, generalizing, and representing relations between quantities and unknowns.

Recommendation 35: mental methods

Many respondents to the ACME survey noted that the draft should give more attention to the development of mental methods – highlighting the connections between strong mental strategies and formal methods – and ACME agrees that the existing references to mental methods need to be enhanced.

It is tempting to assume that mental calculation is a primitive precursor to written algorithms or should be thought of as a last resort – that the written approach is the 'proper' way to tackle a problem. The objective of arithmetic should be effective and efficient calculation – it is easy to construct examples of where a suitable mental method is more efficient than a written approach, and this flexibility needs to be explicitly encouraged. Mental methods often provide a foundation for algebraic reasoning.

The curriculum should be revised to include further guidance and exemplification of mental methods

Recommendation 36: data handling

The profile and expectation of data handling is too low for an area of such critical importance to a 21st century society. This is a rich area for the development of approaches to genuine problem-solving.

The data handling cycle begins with determining what is to be investigated, thus giving a purpose to collecting data, collating, representing it and interrogating and interpreting the results. The draft curriculum does not emphasise sufficiently, or early enough, the importance of all of these stages, and particularly the development of them through discussion, focusing instead on the procedural aspects of organising and representing data. Extensive discussion of the strengths and limitations of different approaches would lay down foundations for good understanding of more sophisticated techniques in the secondary curriculum. Opportunities for children to work with real (rather than contrived) data should be emphasised, and the power of ICT to facilitate this acknowledged. As noted above, primary teachers are well placed to make links with other areas of the curriculum, such as science or geography, and in general knowledge about the world about them: these are all data-rich areas. Additionally, cross-curricular opportunities to develop ideas of chance and of risk should be capitalized on: these again lay foundations for later more formal work with probability.

There is a clear opportunity to make the data handling strand of the curriculum more challenging and we recommend that the Department considers this carefully.

The Department should ensure that the mathematics curriculum is developing pupils in parallel with the science curriculum, and monitor the outcomes of the current Nuffield-funded study on the teaching and learning of probability in Primary schools, to ensure practice in this area is as well-informed as possible.

ACME August 2012

ACME Advisory Committee on Mathematics Education

Appendix A

Suggested rewordings of the aims

Aims

The National Curriculum for mathematics aims to ensure all pupils:

- become **fluent** in the fundamentals of mathematics so that they are efficient in using and selecting the appropriate written algorithms and mental methods, underpinned by mathematical concepts
- can **solve problems by** applying their mathematics to a variety of problems with increasing sophistication, including in unfamiliar contexts and to model real-life scenarios
- can **reason mathematically** by following a line of enquiry and develop and present a justification, argument or proof using mathematical language.

Rewording or tweaking of current aims

- First aim:
    - Rewording suggestion: Become **fluent** in mathematical techniques, methods, recall of number facts and algorithms and have a good understanding of the underlying concepts. [Consultation no. 27]
    - The phrase 'selecting the appropriate written algorithms and mental methods' should stress the vital skill of being able to choose the most appropriate depending on the problem. [Consultation no. 16, 41, 49, 56, 92]
    - The phrase 'selecting the appropriate written algorithms…' should be replaced with 'an appropriate written algorithm …' as there are often different strategies for the same process. [Consultation no. 50, 76, 87]
    - Pupils need to 'select and use', not 'use and select'. [Consultation no. 49 and 51]
    - It is not always possible to 'select and use', for example if there is only one possible choice/response. [Anonymous]
    - The phrase 'they are efficient in …' should be given less importance. [Consultation no. 93]

- Second Aim:
    - 'Model real-life scenario' could be changed to 'use their mathematics for a real purposes in school'. [Consultation no. 51]

    - This aim is not explicit enough. The phrase ‘variety of problems’ is insufficient to describe the whole range of different types of problem solving. [Consultation no. 46, 51 and Anonymous]
    - Add ‘often’ to ‘model real-life scenario’. [Anonymous]

- Third Aim:
    - The third aim needs an example to illustrate the meaning of ‘by following a line of enquiry’. [Anonymous]
    - This should be tweaked to include the important step of making and declaring relevant assumptions. [Consultation no. 87]
    - The third aim should include a statement about conceptual understanding. [Consultation no. 87, 92, 99]
    - The third aim should include algorithms too. [Consultation no. 3]

Aims to Add:

- Schools should ensure that pupils enjoy mathematics and therefore are keen to pursue it further.[Consultation no.18]

Suggested aims from the ATM and MA primary group joint response (please see the full response for more details and comments)

The National Curriculum for Mathematics aims to ensure that all pupils:

- *Can reason mathematically by following a line of enquiry and develop and present a justification, argument or proof using mathematical language.*
- *Can solve problems by applying their mathematics to a variety of problems with increasing sophistication, including in unfamiliar contexts and to model real-life scenarios.*
- *Develop conceptual understanding alongside fluency and efficiency in mathematical techniques and procedures with mental methods as a first resort.*
- *Enjoy and feel confident about mathematics, persevere with challenges and demonstrate resilience, flexibility, enthusiasm and curiosity when learning and using mathematics.*

General Comments:

- The aims should emphasise the need to understand the key concepts in order to ensure fluency [Anonymous].
- The aims should refer to the need for children to develop a secure mental basis [Anonymous].
- Certain aspects of mathematics should be mentioned explicitly in the aims: exploration, problem-solving, discussion and practical experience. It should also mention: children enjoying mathematics and gaining a full understanding in order to make sense of what they are doing and why. [Consultation no. 50]
- Fluency should be an aim for all aspects of mathematics, not just calculation. [Consultation no. 51 and Anonymous (2)]
- 'Pupils should therefore be taught to practice and then apply their mathematics': this sentence suggests the two processes are learned in this order when the two processes actually happen together. [Consultation no. 54]
- It is not clear whether these are the aims for Key Stage 1 and 2 or for all key stages. [Consultation no. 82]
- There is a need to explain to all (parents, teachers, employers, general public, etc.) the reasoning behind the aims. [Anonymous]
- There should be a reference to the connectivity of mathematical concepts in the aims. [Consultation no. 53, 84 and 85]
- The aims should refer to students' enjoyment of mathematics and to students developing confidence and the will to persevere. [Consultation no. 31, 34, 35, 36, 60, and 90]
- The aims should refer to the need to learn to communicate mathematically. [Consultation no. 28]
- The aims should refer to attitudes to mathematics, as they do in the English Curriculum. [Consultation no. 19]
- Will parents understand the meaning of the word algorithm? [Consultation no. 49]
- The aims should mention the beauty of mathematics and its importance as a subject on its own and not just as a service subject. [Consultation no. 84]
- The word 'understanding' is missing from the aims. [Consultation no 12, 31, 34, 35, 36 and 53]

Appendix B

Location	Reference	Excerpt	Commentary
Year 1 - Number	**Heading**	Number	• Suggest that this should be replaced by 'Number and Quantity', to stress the origin of number as 'counting something' or 'measuring something' and to reduce the implication that counting should be done in the abstract. (Workshop)
Year 1- Number - Number and place value	**1**	Suggestion: statement to be added:	• Extra statement to be added after **[1],** '*Children should develop an understanding of number: verbally, visually and symbolically'*. This is to make sure that pupils recognise and understand numbers. (Workshop)
Year 1- Number - Number and place value	**1**	Identify using objects and pictorial representations and use the vocabulary of: equal to; more than; less than (fewer); most; least	• This should go further and state that Year 1 pupils should also be taught to use the symbols =, < and > to express and recognise mathematical relationships throughout. (Consultation no. 81) • We would not reasonably expect pupils to learn about 'identify using objects and pictorial representations and use the vocabulary of: equal to; more than; less than (fewer); most; least **[1]**' before they learn the number names or how to count objects. (Consultation no. 99)
Year 1- Number - Number and	**2**	Count from 0 to and across 100, forward and backwards, beginning with 0 or 1, and from any given number	• We wouldn't count from 'zero' but 'one' if we are reciting the numerals or number words

place value			and we are using positive whole (or natural) numbers, not integers or digits. Counting from 1 is more useful in counting a collection. Counting from any number supports addition. Counting backwards from 100 or any other number is needed to support subtraction. (Consultation no. 99)
Year 1 - Number - Number and place value	**3 (and all through the document)**	Count, read and write numbers to 100 in numerals, count in different multiples including ones, twos, fives and tens	• Terminology of 'ones' rather than 'units' is helpful but needs an explanation in the Notes and Guidance of the rationale for changing this widely used terminology. (Consultation no. 49 and 68) • Does this mean the sequence of numbers in multiples or does it mean to have a conceptual understanding and be able to apply counting in two, fives and tens? (Consultation no. 90) • This statement is repeating part of the previous expectation **[2]** and is trying to combine three different objectives which should remain separate. It confuses counting objects, reciting number sequences, and reading and writing numerals. It should be '*Read and write numerals first to 20 and then beyond*' and '*Count in steps of different sizes including 2s, 5s and 10s*' as separate objectives. Children rushed into counting 'in tens' before counting in ones is firmly established leads to miscounting, where children confuse teens and tys for example 10, 20, 30, 14, 15 ….. It is too ambitious for

			most children and will have little or no meaning. (Consultation no. 99)
Year 1- Number - Number and place value	**4**	Given a number, identify one more and one less	• It would be more accurate to say '*given a number, say the number that comes before and after*' otherwise teachers will confuse saying the number sequence and counting one more or one less (or fewer) which is a different skill. (Consultation no. 99)
Year 1- Number - Number and place value	**5**	Recognise odd and even numbers	• This is an important skill but it would be better for pupils not just to 'recognise' but to also be able to recite odd and even sequences, (fits with counting in twos at **[3]**) In what range should they be able to do this? 20 would be reasonable for Y1. (Consultation no. 99) • Recognising odd and even numbers is important for multiplication and division, so thought need to be given to where this statement should be placed in the curriculum. (Workshop)
Year 1- Number - Number and place value	**6**	Read and write numbers from 1 to 20 in numerals and words	• This statement raises a number of questions: *1*- Where is 0? *2*- How does writing eighteen fit with the phonics expectations of Y1? *3*- Putting an emphasis on numbers to 20 means focusing on the hardest numbers (teens) and ignoring the numbers children can make sense of which will help with the teens. (Consultation no. 70) • This statement is a repetition of part of **[3]** and

			therefore partly redundant; is it reasonable to expect a six year old to write the numbers ten to twenty as words? Does it make them better at counting? Is it mathematics? It is more important for them to be able to count and use the numbers than write them as words. (Consultation no. 99)
Year 1 – Number- Number and place value	**7**	Distinguish between and use ordinal and cardinal numbers.	• The emphasis should be on 'using' rather than 'distinguishing'. The words cardinal and ordinal should be explained in more details, as they will have a different meaning for teachers than for mathematicians. (Workshop) • We might want teachers to '*distinguish between*' but we want pupils to be able to '*use ordinal and cardinal numbers correctly*'. (Consultation no. 99)
Year 1 – Number- Number and place value	**1 to 7**	**Suggestion from Consultation no. 99 to replace all 7 statements with:**	• *Learning to recite number sequences forwards and backwards to 20 then 100.* • *Learning to say the number before and after a given number in the range of 1 to 20 and then beyond.* • *Recognising, identifying and sequencing and ordering numerals in the range of 1 to 20 and beyond.* • *Read and write numerals to 20 and beyond.* • *Locate numbers on the number line/track first*

			to 20 then to 100. • *Learning to recite number sequences forwards and backwards in 2s, 5s and 10s including odd and even numbers.* (Consultation no. 99)
Year 1 – Number- Number and place value	**8** **Also 17, 45, 55, 69, 70, 88, 95, 104, 144, 194, 208, 222, 247**	Ensure pupils practise counting in ones, twos, fives and tens from different multiples to develop their recognition of patterns in the number system.	• Over reliance on counting and memorising as a means of instruction. (Consultation no. 73) • The recognition of patterns in the number system could be enhanced through the use of calculators and ICT. (Consultation no. 87) • This does not emphasise the importance of backward counting. In addition, reciting alone will not develop children's 'recognition' of patterns without also identifying, recognising, sequencing and ordering numerals (Wright, 2006). Note that identifying and recognising numerals are separate from what is sometimes called 'interpreting numerals'. The latter is properly a part of learning place value. (Consultation no. 99)
Year 1 – Number- Number and place value	**9**	Ensure pupils begin to recognise place value in numbers beyond 20 by reading, writing, counting and comparing numbers up to 100.	• In year 1, place value is mentioned in the Notes and Guidance but not in the programme of study. This does not appear to suggest a need for pupils to have a sound grasp of place value in the context of base 10. (Anonymous) • Children do not learn about the 'place value' code simply by studying patterns in numerals.

			They need actively to organise physical objects to show through action how grouping into multiples of tens underlies our conventional system for generating number names. (Consultation no. 6) • Conceptual place value also requires locating numbers on a number line and making numbers with materials such as bundles of straws to establish an understanding of quantity (Wright, 2012). Also important is learning to increase and decrease numbers by ones, tens and hundreds and progressing this topic by working firstly with materials and ultimately with numbers only (that is, without materials). Pupils will not recognise place value by simply reading, writing and comparing numbers. (Consultation no. 99)
Year 1 – Number – Number and place value- Notes and guidance	**10**	Ensure pupils are taught when and how to use numbers for ordering (e.g. first, second, third), for counting (1, 2, 3) or to indicate a quantity (e.g. three apples, 2 centimetres). Exclude the terms “ordinal” and “cardinal”.	• The EYFS Curriculum uses the heading ‘*Numbers as labels and for counting’*, It would be useful for the Notes and Guidance to make links to this and to give an example of ‘labelling’. The last sentence appears to be a remnant of version control and is unnecessary. (Workshop) • The wording contradicts itself. If this statement is referring to ensuring pupils have an understanding of using numbers for counting, one to one correspondence (cardinality) in addition to ordering objects (ordinality), then it

			should be made clear. (Consultation no. 99)
Year 1 – Number – Addition and Subtraction	**11**	Read, interpret and practise writing mathematical statements involving addition (+), subtraction (-) and equals (=) signs accurately	• The terminology of 'mathematical statements' is unclear. Are these recorded vertically, horizontally, as words, as symbols? (Consultation no. 49 and 68) • This statement should make it clear that the aim is for children to begin to understand these operations, building on the mathematical understanding that many children have already grasped when they start school. (Consultation no. 7) • It is particularly important to make sure that students understand the meaning of ' =' as this is often misunderstood as meaning 'is the answer' There should be an explanation in the Notes and Guidance. (Consultation no. 65 and 93) • Children should also be taught the symbols < and >. If they don't, it could hinder their understanding of mathematical equivalence without enhancing their understanding of arithmetic operations. It would better to introduce these symbols before the operator symbols are introduced. (Consultation no. 81) • Children need to learn how to add and subtract and solve addition and subtraction problems before practising writing mathematical statements. (Consultation no.

			90) • The signs +, -, = are mathematical shorthand. They are put in place only after understanding of the operations has developed through practical experience and discussion. There is no mention of the ways in which children develop more sophisticated calculation strategies. Teachers need to be aware of the relevant strategies and how to support young children in developing them. There is no mention of the essential building blocks of sophisticated non-count-by-one strategies for addition and subtraction, structuring numbers from 1 to 10 and then 1 to 20. (Consultation no. 99)
Year 1 – Number – Addition and Subtraction	**12**	Add and subtract 1-digit and 2-digit numbers to 20 (9 + 9, 18 - 9), including zero	• These kinds of statement are also relevant for pre-algebra. (ACME) • This could be rephrased to include the expectation that children should also understand these processes. For example, the statement could be: '*Children should be taught to understand the concept of addition as combining two or more sets to make a total and understand the concept of subtraction as 'take away', 'how many more to make' and difference' in order to 'add and subtract 1-digit and 2-digit numbers to 20 (9+9, 18-9), including zero*.' (Consultation no. 49 and 68)
Year 1 –	**14**	Recall and use number bonds and related subtraction facts within 20	• This should be until 10, not 20. 20 should be

Number- Addition and subtraction			in year 2. (Workshop) • Bonds to 10 is realistic for all and to 20+ for some. (Consultation no. 65) • Committing facts to memory is the final step, not the initial step. (Consultation no. 99)
Year 1 – Number- Addition and subtraction	**15 (and others)**	Solve simple word problems that involve addition and subtraction.	• This reads as if problem solving is always a 'add on'; applying the skills having learned them. Real problem solving is about 'messy' problems, people approaching solutions in a different way, not necessarily reaching a solution, etc. (Consultation no. 16) • Children need understanding in order to make use of 'operations' in problem solving situations. (Consultation no. 99)
Year 1 – Number- Addition and subtraction	**11 to 15**	**Suggestion from consultation no. 99 to replace statements 11 to 15 with:**	• *Counting items in collections and rows, including temporal sequences and patterns, counting screened collections.* • *Counting on and back to solve addition and subtraction problems.* • *Recall and use addition and subtraction number bonds first to 10 then 20.* • *Understand addition can be done in any order but subtraction cannot.* • *Begin to use addition (+), subtraction (-) and*

			equals (=) signs accurately. • *Solve simple word problems that involve addition and subtraction.* • *Beginning to count in tens and ones using bundles and sticks or similar.* (Consultation no. 99)
Year 1 – Number- Addition and subtraction	**16**	Ensure pupils practise reading and writing mathematical statements regularly so that they become fluent.	• Statements such as this emphasise the teaching procedure at the expense of children making sense of the mathematics they are learning. (Consultation no. 99)
Year 1- Number – Addition and subtraction- Notes and Guidance	**17**	Ensure pupils practise so that they memorise their number bonds to 20 in the three forms (e.g. 9 + 7 = 16; 16 – 7 = 9; 16 – 9 = 7), and that they can record their answers. This will prepare them for Year 2 when they are taught how to add and subtract two 2-digit numbers.	• This statement is also relevant for pre-algebra. (ACME) • A fourth form should be added: '7+9=16'. (Workshop) • If teachers are having choice about yearly content, this should be amended to say '*this will prepare them for later addition and subtraction of 2-digit numbers'*. However, if there are going to be statements that connect ideas through the curriculum there should be many more of them; by and large these vertical connections are not explicit. (ACME) • Statements **[12], [14],** and **[17]** are all about similar material, so if **[12]** and **[14]** are moved to year 2, so should **[17]** be. (ACME)

			• This is welcomed as it helps reinforce a proper understanding of symbolic equivalence. (Consultation no. 81) • The phrase 'to prepare them for the next year' is used frequently. We feel this phrase devalues that year's work. (Consultation no. 94)
Year 1- Number- Multiplication and Division	**18 to 20**	Recognise and write the multiplication symbol (x) and the division symbol (÷) in mathematical statements, calculating the answer with the teacher using concrete objects [18] Solve word problems involving simple multiplication and division, with teacher support [19] Ensure pupils are introduced to the multiplication (x) and division (÷) symbols so that they can recognise and write them accurately. They should distinguish them from addition and subtraction. This prepares them for Year 2 when they are taught how to multiply and divide using two 1-digit numbers with concrete objects and then using numbers within the multiplication table [20]	• The introduction for multiplication and division should not focus first upon the written symbols of those operations. (Consultation no. 6, 56, 82 and 90) • The 'repeated addition' meaning of multiplication must wait until year two, partly to provide space for addition and subtraction to take root without confusion and partly because multiplication' as repeated addition is a contentious approach to multiplication. When it is introduced in Year 2, it should not be via the symbol 'x' but via the conceptual notion of repeated addition. This should be linked with 'counting on in jumps' and the number line and informally with the inverse (breaking a given quantity into groups, or sharing). The symbol 'x' should only be used as shorthand when the calculations are relatively secured. (Workshop) • There should be a mention of practical 'groups of' and 'sharing'. The act of repeated grouping and sharing should be taught alongside the

			symbols, if not slightly before. (Consultation no. 53) • This needs to be more specific about the concepts behind the operations. It could be added in the Notes and Guidance. (Consultation no. 65) • The introduction of division in Year 1 is too soon. (Consultation no. 94) • The curriculum often moves from 'concrete', for example using objects for counting, through to more abstract work. Young children can use their imagination and don't always need concrete apparatus to enable them to think. Conversely, older pupils and even adults can sometimes find apparatus useful to support understanding. (Consultation no. 82) • As with addition and subtraction, understanding of the operations has to be developed through practical experience and discussion before symbols are introduced. Early multiplication and division strategies are developed by children calculating the answer using advanced additive and subtractive strategies not by counting concrete objects. Also, as with addition and subtraction, children need understanding in order to make use of 'operations' in problem solving situations. (Consultation no. 99)

Year 1- Number- Multiplication and Division	**18 to 19**	**Suggestion from consultation no. 99: Statements 18 to 19 should be replaced with:**	• *Recognise and use spatial patterns and finger patterns to make combinations first to 5 and then 10.* • *Doubles and halves to 10 and beyond.* • *Use combinations and partitions of numbers in the range to 5 then 10 and 15.* • *Use number sequences for 2s, 5s and 3s to solve multiplicative tasks.* • *Using equal grouping and sharing to solve simple multiplication and division problems using manipulatives and arrays.* • *Solve simple word problems involving simple multiplication and divisio*n. (Consultation no. 99)
Year 1- Number- Multiplication and Division	**20**	Ensure pupils are introduced to the multiplication (x) and division (÷) symbols so that they can recognise and write them accurately. They should distinguish them from addition and subtraction. This prepares them for multiplying and dividing using two 1-digit numbers with concrete objects and then using numbers within the multiplication tables.	• Multiplication is introduced as repeated addition. Addition would therefore need to be sufficiently robust before one can make sense of multiplication - yet the two themes appear together in Year 1. (Consultation no. 79 [LMS response]) • Children should be recognising links between the four operations rather than distinguishing between them. (Consultation no. 16) • This does not support what we know about how understanding of the operations has to be

			developed through practical experience and discussion. (Consultation no. 99)
Year 1- Number- Fraction	**21-22-23**	• Recognise, name and write ½ as one of two equal parts of an object, shape or quantity [21] • Recognise, name and write ¼ and ¾ as parts of an object, shape or quantity [22] • Find ½, ¼ and ¾ of a shape or quantity. [23]	• Note that the comments about fractions, decimals and numbers are being reported as they were given to us, but what is really required is thorough coherent work on progression in all aspects of number. (ACME) • It would be better to teach this in year 3 and 4. (Consultation no. 11) • Halving a unitary object is a much simpler concept than halving a quantity. The latter requires notions of quantity and division. This should be moved to Year 2. (Consultation no. 11) • There should be more emphasis on finding fractions practically in a range of contexts. (Consultation no. 41) • Finding a fraction of a quantity relates to concept of multiplication and division beyond this level in the written programme of study. (Consultation no. 43) • There seems to be a discrepancy in progression. The expectation in Y1 is that children can find ¾ of a quantity, which is quite challenging but in Y2, they are recognising, naming and writing fractions and counting in halves and quarters **[67 and 68],**

			which is a lot easier. (Consultation no. 56) • Fractions do not rely on counting to express their meaning as relational quantities and a heavy emphasis on counting parts of an object or shape can lead to very limited beliefs as to what fractions are (Gould P. in Wright et al 2012). Unless children have a good understanding of division they would find this difficult. Early work linking halving and doubling would be more appropriate. (Consultation no. 99)
Year 1- Number- Fraction	**21 to 23**	**Suggestion from Consultation no. 99 : Statements 21 to 23 should be replaced with:**	• *Finding halves of objects and quantities and understanding 'equal' halves.* • *Finding quarters of objects and quantities by 'halving a half'.* • *Relate fractions to doubles and halves and division by equal partitions..* (Consultation no. 99)
Year 1- Number- Fraction	**24**	Ensure pupils are taught ½, ¼ and ¾ as objects and then as operators on discrete and continuous quantities. As objects, pupils recognise and combine ½, ¼ and ¾ as parts of a whole. As operators, pupils recognise and find, for example, half of a length, container, set of objects and shapes.	• This is to establish multiple meanings for fractions with numbers and quantities, as a sound preparation for ratio later, but 'container' does not make sense. It is an attempt to describe half of a liquid quantity, i.e. half the contents of a container. But 'half a container' does not really help anyone's fraction understanding. The 'half ... shapes' probably means 'half a shape' such as half a circle, half a square, i.e. a model for half using

			area. It should really read '*half a length, quantity, set of objects, or shape'*. (Workshop) • When abstract ideas are presented prematurely to children – as in the case of 'fractions as objects' in KS1 (statement **[24])** – this approach is self-defeating; the later 'practice' effectively reinforces poor understanding and encourages empty ritual. The proposed work on 'fractions as objects' for years 1 and 2 (statements **[24], [69], [70])** would result in empty verbal rituals quite unrelated to any of the stated aims of the curriculum. Such abstract ideas can safely be left until KS2 when – with suitable imagery – children can begin to *generalise* their way to 'fractions as objects' on the basis of extensive and contextualised experience with 'fractions as operators'. 'Fractions as operators' within appropriate contexts in KS1 are perfectly appropriate – indeed necessary. (Consultation no. 6) • Fractions should not be listed officially in Year 1. The language could begin to be used in Year 2 (half, quarter, three quarters). The notion of unit fractions could become clearer in Year 3 (splitting a whole into equal parts) with the inclusion of thirds and fifth to match the emerging tables. Combining fractions with the same denominator arises naturally after year 3 and is best listed in Year 4.

			(Workshop). • Naming and writing ¼ and ¾ as fractions is inappropriate in terms of cognitive demands and doesn't link with doubling and halving or division. (Consultation no. 90)
Year 1- Geometry and Measures- Properties of shapes	**25-26**	• Recognise and name common 3-D and 2-D shapes, including: - 2-D shapes (e.g. square, rectangle, circle and triangle) - 3-D shapes (e.g. cube, pyramid and sphere). [25] • Ensure pupils practise regularly using and naming common 2-D and 3-D shapes and related everyday objects, so that they recognise the properties irrespective of their orientation or size.[26]	• There are category errors here. In geometric reasoning you have to develop an appreciation of the difference between (i) everyday language and precise language (ii) reasoning from visual appearance and reasoning from properties. i. Squares ARE rectangles so should not be listed with the same status. ii. Circles and squares are shapes which always look the same, indeed are the same shape. Circles look the same in any orientation but vary in size. Squares need to be recognised in any orientation and size. iii. Rectangles (including squares) and triangles can be an infinite number of different shapes, as well as varying in orientation and size, so for them line **[26]** should include the words 'orientation, size and shape'. iv. Cubes, spheres and pyramids have the same problems, spheres varying only in size; cubes in orientation and size,

			pyramids in shape, orientation and size. For some reason cuboids, the 3-D equivalent of rectangles, are not listed. (Workshop) • Children should also have a range or practical experiences with shapes which go well beyond naming. This would build on the recommendations of a recent review of research on children's understanding of shape space and measures (Bryant 2009) which reports that children enter school with impressive informal knowledge about shape and space and that an important challenge for mathematics education is how best to harness this implicit knowledge (page 3). (Workshop) • Properties of shapes at **[25]** does not build adequately on from the expectations in EYFS; where are the exploring with shapes, the understanding of the properties of shapes, flat and curved faces, and problem solving with shapes which can build and roll? (Consultation no. 99)
Year 1 - Geometry and measures- Position, Direction, Motion	**27 and 29**	Describe position, directions and movements including half, quarter and three-quarter turns.[27] Ensure pupils practise making turns to show they can understand half, quarter and three-quarter turns and routinely make these turns in a clock-wise direction. This will prepare them later when they are taught about angles. [29]	• Turns and the measure of turn are difficult concepts to understand. Three quarters is far more conceptually demanding than half (or quarter – which would be better left to Y2). (Workshop) • This is an example of the need to tie up connections between fractions as number and

			other occurrences of fractions. (ACME)
Year 1- Geometry and Measures- Measures	**30,31, 32,33, 35, 36, 37**	• Compare, measure and record the following using standard units for: - lengths and heights (e.g. long/short, longer/shorter, tall/short, double/half) - lengths and heights (metres, centimetres) - mass (grams, kilograms) - capacity and volume (litres) - time (hours, minutes, seconds) [30] • Compare, describe and solve practical problems for: - lengths and heights (e.g. long/short, longer/shorter, tall/short, double/half) - mass (e.g. heavy/light, heavier than, lighter than) - capacity and volume (full/empty, more than, less than, quarter, three quarters full or empty) - time (quicker, slower, earlier, later) [31] • Recognise and use pounds (£) and pence (p) with different denominations of money, including coins and no.tes [32] • Tell the time to the hour and half past the hour [33] • Recognise and use the language of dates, including days of the week, weeks, months and years. [35] • Ensure pupils become proficient in using measuring tools, such as a ruler, weighing scales and containers to become familiar with standard measures. [36] • Ensure pupils regularly practise reading and writing units of measure accurately and become familiar with the correct terminology and related symbols. This supports work in Year 2 when pupils are taught to calculate sums and differences with these units. [37]	• This is a large proportion of the current KS2 measures curriculum by the age of 5. Dates are conceptually challenging and it is not realistic to expect 5 year olds to be able to understand them fully at this stage.– The conceptual development of these statements requires work on comparison, ordering, conservation (as suggested in **[31 and 34])** and the use of non-standard measures. The use of measuring tools should be in the context of practical problem solving rather than to become familiar with standard measures. **[36]** (Workshop) • It would be better to reverse statement **[30]** and **[31]** so that direct and indirect comparisons of objects are shown to precede both the introduction of units of measure in general and of standard units in particular. (Consultation no. 6) • **[30]** The expectations are too high, the use of standard units is inappropriate. Young children would not be able to deal with the size of the conversion numbers; they are too large. We need children to be secure in measuring and comparing with non-standard units so that they can recognise the need for standard units before being introduced to them. (Consultation no. 99)

			• Bryant (2009: 5) stresses the importance of the conceptual basis of measurement and not just the procedures (page 5). Children should work not just with standard units but also with familiar objects like cups and hands (ibid: 6). (Workshop) • It would be good to include non-standard units with standard units. Problem solving with non-standard units allow children to realise the need for standard units of measure and develop a relational understanding of size and benchmarking. (Consultation no. 56, 60, 86, 90 and 93) • There are two aspects of time. 1) Understanding what time is and what are the units for measuring it and 2) reading the clock. Using the instrument does not develop conceptual understanding. Reading the analogue clock requires reading of a different scale for each hand – well beyond the stage of year 1 and 2 pupils. Suggest we just talk about the passing of time in Y1 and leave 'telling the time' until later years. Y2 could talk about units for measuring time (hours, week, months, minutes) and pupils should know there are 60 minutes in an hour, 7 days in a week, etc. (Consultation no. 11) • Digital clock reading is introduced years later than analogue, although it is 'easier' to read the time from a digital display than from an

			analogue clock face. (Consultation no. 82) • Seconds and years are not appropriate units of measure for children who are only 5-6 years old; they struggle to estimate the size of these quantities of time **[35].** (Consultation no. 99) • Children usually learn about capacity to start with and not volume. (Consultation no. 16 and 99) • The difference between capacity and volume should be made clear. The guidance should be more specific about this. (Consultation no. 11 and 56) • Capacity should be introduced with practical equipment such as sand and water. (Consultation no. 90) • Coins are symbolic (e.g. a 10p piece does not look like ten pennies in size or shape, so it has to be understood that 'it stands for' the same value as ten pennies) – this is more conceptually challenging for children than visible relations between quantities. (Workshop) • The understanding of money using pounds and pence is reliant on a very secure understanding of place value. (Consultation no. 60) • A scale that was graduated only in kg or in g

			• Statement **[37]** suggests that Year 1 children can easily learn to spell 'centimetres', a word that is phonically irregular. (Consultation no. 92) • Statement **[37]:** A statement that assumes a year by year curriculum – it could be reworded to refer to progression. (ACME) • This does not match **line [6],** since children are only being expected to write and read numbers up to 20. Lines **[30]** and **[32]** would only be fully possible with greater numbers. (Workshop and Consultation no. 12) • At year 1, pupils do not have the manual dexterity to use rulers and scaled containers at **[36]** accurately. We know they struggle with seeing that length involves measuring space rather than just counting units. (Consultation no. 99)
Year 1- Geometry and Measures- Measures	**36**	Ensure pupils become proficient in using measuring tools, such as a ruler, weighing scales and containers to become familiar with standard measures.	• We measure mass using a balance. Year 1 pupils do not read or use scales, they use a balance. Bathroom scales measure weight: balances always measure mass as they eliminate the effect of gravity. If we want to measure mass, we must use balance.

			(Consultation no. 11)
Year 1- Geometry and Measures- Measures	**37**	Ensure pupils regularly practise reading and writing units of measure accurately and become familiar with the correct terminology and related symbols. This supports work in Year 2 when pupils are taught to calculate sums and differences with these units.	• The word 'sum' should be used in relation to addition, whereas in this circumstance it has been used instead of 'number sentences', which relate to addition and subtraction. (Consultation no. 7)
Year 1- Geometry and Measures- Measures	**38**	Ensure pupils regularly practise the vocabulary of time, including telling the time throughout the day, first using "o'clock" and then "half past".	• Why not make the link between analogue and digital from the start? This statement could be moved to Year 3 and children could be asked to tell the time as 7 o'clock and 7:00, half past 7 and 7:30. (Consultation no. 11)
Year 2 – Number- Number and Place Value	**39**	Read and write numbers to at least 100 in numerals and in words	• It is not necessary for children to write the words up to 100. (Consultation no. 99)
Year 2 – Number- Number and Place Value	**40**	Recognise the place value of each digit in a 2-digit number (tens, ones)	• The word 'recognise' does not imply that the pupils should be able to construct numbers using an understanding of place value but only respond to 'what is the number worth' It would be more helpful to use '*compose and decompose numbers*' which would reflect the skills required for a better understanding of number structures. (Anonymous) • We would want children to have experience of 2 and 3 and even 4 digit numbers in Year 2 if they are ready for them but certainly go beyond 100. (Consultation no. 99)

Year 2 – Number- Number and Place Value	**42**	Compare and order numbers from 0 up to 100; use <, > and = signs	• The symbols '<' and '>' are not appropriate for year 2. Concepts should be developed in tandem with symbols. Introducing symbols first will cause confusion. (Consultation no. 82)
Year 2 – Number- Number and Place Value	**44**	Solve word problems using place value and number facts with increasing precision.	• This statement could include a reference to 'generalising' - in terms of understanding place value enough to generalise about the number system. (Workshop) • As in many other statements, this would benefit from rewording 'word problems' to include a more useful view of problem-solving. (ACME + Consultation no. 11)
Year 2 – Number – Number and place value	**47 and 152**	• Ensure pupils are fluent and apply their knowledge of larger numbers to discuss and solve problems that emphasise the value of each digit in 2-digit numbers. For example, they should read 46 as 'forty and six' and solve addition and subtraction mentally such as, 36 – 6 = 30 and 50 + 6 = 56. [47] • Ensure pupils say and write the numbers correctly and with precision, so that they are clear about place value and confident when working with mental calculations. This will prepare them for Year 5, when pupils are taught to calculate the sum and difference of two decimal numbers (up to 2 decimal places).[152]	• This is ambiguous: children should not *read* 46 as 'forty and six' (followed by 'forty and seven'...). This should be replaced by 'Children should *interpret* 46 as 'forty and six' when calculating'. (Consultation – no. 54) • A statement that assumes a year by year curriculum – it could be reworded to refer to progression. Also why are children being prepared in year 2 for something in year 5? (ACME) • **[47]** is a rather abstract way of seeing place value rather than an additive way; it relies on naming rather than making, breaking up and incrementing and decrementing for example 200 + 40 + 6. (Consultation no. 99)

Year 2 – Number – Addition and Subtraction	**48**	• Rapidly recall and use addition and subtraction facts to 20	• This statement relies on the children habituating and memorising the facts to 20, not developing an understanding of the structure of numbers to 20 through combining and partitioning. (Consultation no. 99)
Year 2 – Number – Addition and Subtraction	**49 and 56**	• Add and subtract numbers with up to two 2-digits including using column addition without carrying and column subtraction without borrowing. [49] • Ensure pupils practise column addition and subtraction to write numbers with precision to calculate answers. This also reinforces the concept of place value. Horizontal written methods should progress rapidly to more efficient column methods to help prepare pupils in Year 3 when they are taught column addition with carrying and subtraction with borrowing. [56]	• This is not informed by research and undermines work on mental strategies **[50].** Research suggests that presenting calculations 'horizontally' encourages children to think about the numbers being used and 'column' methods don't do this. (Consultation – no. 12, 23, 31, 34, 35, 36, 47, 48, 54 and 70) • The National curriculum has never prescribed the written procedures to be used and most other countries don't do this either. Note that column methods aren't always the most efficient e.g. 998+52 or 2004-7. (Consultation – no. 54) • This does not seem to be supported by research – we would suggest that prescription regarding methodology is left as notes or examples, and that end of year objectives focus on whether children are able to give the correct answer to a problem, and whether they understand any processes they are applying to find those answers. (Consultation no. 72) • What are column methods? Potentially there is a very wide interpretation of this term e.g. expanded or compact; subtraction could be

			decomposition, equal addition or complimentary addition. (Consultation no. 19, 46, 49 and 68) • Unsure about the introduction of column methods for addition and subtraction in Year 2. Before written methods are introduced, a thorough understanding of what the numbers are, how they can be partitioned and manipulated on models such as a number line and a proper grasp of estimation and what the answer should roughly be must be embedded. This must be secure before using formal written methods which children can use by rote but may not have embedded understanding of what they are actually doing. (Consultation no. 14) • The terms 'borrowing' and 'carrying' have largely been replaced by the more meaningful terms 'partitioning' and 'exchange' for decomposition in subtraction. (Consultation no. 4, 6, 49, 51, 68, and 78) • This statement **[49]** could lead teachers to think that decomposition is the preferred method of subtraction, but this may not be the case. (Consultation no. 93) • **[56]** assumes a year by year curriculum – it could be reworded to refer to progression. (ACME) • Pupils should not be encouraged to use

			column format for 2-digit addition and subtraction as these calculations are more effectively done mentally. (Consultation no. 92) • Limiting children to additions that don't involve carrying in Year 2 will mean that it becomes a complication in Year 3. If addition was taught with 'carrying' from the start children would understand the method more fully. For this reason, introducing it in Year 2 is flawed. (Consultation no. 4) • Rapid progress is misguided if children won't have had the time to develop really secure foundations to allow them to understand why the vertical algorithm works. (Consultation no. 16) • There should be more emphasis on equipping children with a flexible range of strategies so they can use the 'right tool for the right job'. (Consultation no. 26) • Expanded column would be okay for + and – with LOTS of work with base ten material and good understanding of place value. This needs to be explicit so it is not taught as a procedure. (Consultation 65) • This goes against the research of Ian Thompson and Realistic Mathematics Education (RME) and what has been accepted by those involved in mathematics

			education for the last 20 years. We should hold off from column addition and subtraction until children have strong mental strategies; they should not be taught together as the children then use column as a single method and don't continue to develop mental methods. (Consultation no. 99)
Year 2 – Number – Addition and Subtraction	**51 and 151**	• Use subtraction in 'take away' and 'find the difference' problems [51] • Ensure pupils continue practising formal written methods and mental methods with increasingly large numbers, and include the terms 'sum' and 'difference'. For mental calculations, include increasingly large numbers, for example, 12,462 - 2,400 = 10,062 or 12,462 + 600 = 13,062**.** [151]	• This statement does not match the use of 'difference' and 'take away' with statement **[151].** The word 'difference' is introduced in year 2, but 'sum' only appears in year 4 **[151].** (Workshop) • **[Line 51]** states two meanings of subtraction. We also need statements about the meanings for addition, multiplication, or division as all these also have multiple meanings in words. (Workshop)
Year 2 – Number – Addition and Subtraction	**52-53**	• Recognise and show that addition can be done in any order (commutative) and subtraction cannot [52] • Recognise and use addition and subtraction as inverse operations including to check calculations [53]	• This is appropriate if they have developed non–counting strategies and understand they can use commutative and inverse to 'undo' addition and subtraction not just 'reverse' number statements. (Consultation no. 99)
Year 2 – Number – Addition and Subtraction	**54**	Solve word problems with addition and subtraction of numbers with up to 2-digits.	• The reference to 'up to two digits' is too limiting – it should be 'at least two digits' or similar. (Workshop) • 'Word problems' could be reworded to expand the meaning of problem-solving to include unfamiliar situations and choice of appropriate

			methods. (ACME)
Year 2 – Number – Addition and Subtraction	**56 and 99**	Ensure pupils practise column addition and subtraction to write numbers with precision to calculate answers. This also reinforces the concept of place value. Horizontal written methods should progress rapidly to more efficient column methods to help prepare pupils in Year 3 when they are taught column addition with carrying and subtraction with borrowing. [56] Ensure pupils continue to practise the use of column addition and subtraction with increasingly large numbers, using carrying for addition and borrowing for subtraction. [99]	• It is not clear from the phrase 'with borrowing' which of the two main 'paper and pencil' methods of subtraction is being advocated, 'decomposition' or 'equal addition'. From the use of the phrase 'with borrowing', it appears to be 'equal addition'. '*Mathematics 5-11 A Handbook of Suggestions*', HMI Series : Matters for Discussion, HMSO 1979 , states that 'equal addition' is not suitable for young children, unless they have had a varied experience of the principle that adding the same number to both numbers leaves the difference unchanged. It can lead to the use of misleading jargon, such as 'borrow and payback'. (Consultation no. 13 and 65) • It would be better to secure longer term understanding rather than 'rapid progress'. (Consultation no. 87) • The mention of 'borrowing' as a subtraction procedure without any understanding of place value or decomposition is not helpful and doesn't support conceptual understanding. (Consultation no. 90) • Column methods should be delayed until children can competently add and subtract mentally with all 2 digit numbers up to 100. (Consultation no. 99)

Year 2 – Number – Addition and Subtraction	**57**	Ensure pupils practise mental addition and subtraction of two numbers of up to 2-digits, with answers not exceeding 100. They should know how to check calculations, including by adding to check subtraction and adding numbers in a different order to check addition; for example 5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5.	• The principles of commutativity and inversion should be developed from KS1 onward. This is hinted at in this statement but should be made more explicit and more pervasive through both Key Stages. (Consultation no. 81)
Year 2 – Number- Multiplication and division	**59**	Recall multiplication and division facts for the 2, 5 and 10 multiplication tables [59] in relation to [67] : Recognise, name and write fractions $^{1}/_{4}$, $^{1}/_{3}$, $^{1}/_{2}$, $^{2}/_{3}$ and $^{3}/_{4}$ of a whole	• This statement doesn't match item **[67].** If you want children to learn ¼ fractions, they should be able to also recall multiplications and division facts for 4, as well as 2, 5 and 10. (Workshop) • It would be helpful to bold the times tables that are 'new' to be learnt each year. (Consultation no. 56)
Year 2 – Number- Multiplication and division	**59-60-61**	• Recall multiplication and division facts for the 2, 5 and 10 multiplication tables [59] • Use the multiplication (x), division (÷) and equals (=) signs to read and write mathematical statements [60] • Write and calculate mathematical statements for multiplication and division within the multiplication tables [61]	• These expectations are not consistent with research. Whilst it is important that young children develop multiplicative reasoning alongside additive reasoning this needs to be done in practical and meaningful contexts. Formal recording should be delayed until children are ready. (Consultation – no. 54) • Starting off teaching multiplication with symbols is not the way into understanding the concept of multiplication. They should develop mental strategies and then progress onto symbols. There is nothing in the guidance to show how children's understanding of

			multiplication and division builds on their knowledge of addition and subtraction. Teachers will find it very difficult to teach these topics in a meaningful way that will enable children to master the concepts. (Consultation no. 99) • The terminology of 'mathematical statements' is unclear. Are these recorded vertically, horizontally, as words, as symbols? (Workshop and Consultation no. 49 and 68) • Some examples for statement **[60]** would be helpful and also some indication of why this contributes to progression. (Workshop + ACME)
Year 2 – Number- Multiplication and division	**62**	Recognise and use the inverse relationship between multiplication and division to check calculations	• This is too sophisticated for that age. It should be more about preparing children to learn about it, rather than expecting them to do it. The statement would be better to read: *Children should explore the inverse relationship....* (Workshop) • Pupils may learn to switch intelligently between multiplication and division. But they don't 'recognise the inverse relationship'. This is too formal. (Consultation)
Year 2- Number -Multiplication and Division	**63**	Ensure pupils can recognise and show that multiplication can be done in any order (commutative) and division cannot	• This should be taught later. Shouldn't use the word '*ensure*', better to say: '*bring to their attention'*. (Workshop) • Note that this refers only to the repeated

			addition meaning for multiplication, or number facts using whole numbers. It is a pre-algebra understanding. (ACME) • Pupils may appreciate that 3x4 and 4x3 give the same answer. But this is not helped by using the word 'commutative'. Learning that division is not commutative will not help children. The word 'commutative' should appear in the guidance and not as a word for children to use at this stage. (Workshop)
Year 2- Number -Multiplication and Division	**65 and 32**	Ensure pupils are taught multiplication and division through sharing out quantities; finding simple fractions of objects, numbers and quantities; doubling numbers and quantities; and find related halves. This also links to recognition of division as sharing and grouping**. [65]** Recognise and use pounds (£) and pence (p) with different denominations of money, including coins and notes. **[32]**	• The beginning of statement **[65]** would make more sense as '*ensure pupils experience multiplication etc....*' (ACME) • Division as 'sharing' should be extended to 'sharing and grouping'. The document should refer to the use of different representations. For example, none of the activities in the first sentence refers to 'grouping'. (Workshop). • This seems to come before line **[32]** in meaning, rather than a year later. (Workshop) • Addition and subtraction may also be acceptably used to multiply and divide in appropriate circumstances. (Anonymous) • We welcome the connection here between fractions and division. (ACME) • It is good to allow teachers the freedom to set reverse multiplication problems (3 x ?? =6)

			and to introduce reverse grouping and sharing alongside multiplication. However formal division should not be obligatory simultaneously with multiplication. In particular, the division symbol should not be listed alongside the multiplication symbol as though they should both appear at the same time. It would be better to teach it in Year 3, to give time for the operations to become secure. (Workshop) • Statement **[65]** is confusing as it implies that you can teach multiplication through sharing out quantities. (Consultation no. 56)
Year 2 – Number- Multiplication and Division- No.tes and Guidance	**66**	Pupils are introduced to the multiplication tables in Year 2. Ensure pupils practise 2, 5 and 10 multiplication tables up to x12 so they are fluent in recalling them. This includes using related division facts to perform written and mental calculations.	• The x12 tables were historically learned because of their use in the Imperial system, which is no longer the case. Emphasis throughout primary should be on tens to embed decimal number system, only up to tens are necessary for mental and written algorithms. (Workshop and consultation no. 12, 43 , 63, 65, 97 and 99) • We should be encouraging children to understand that they can use 10x and 2x to find larger facts. (Consultation no. 47) • The Notes and guidance has elements that should be in Y1. The 4 times tables is needed for finding e.g. ¾ of a larger number and it is logical to teach this table alongside work on ¼. (Consultation no. 53)

Year 2- Number- Fractions	**67**	Recognise, name and write fractions $^{1}/_{4}$, $^{1}/_{3}$, $^{1}/_{2}$, $^{2}/_{3}$ and $^{3}/_{4}$ of a whole	• Fractions should be taught later; they should be introduced in year 2 only as a language. Thirds are also much more difficult for children to comprehend than fourths, it should be introduced later. (Workshop + Consultation no. 54) • It is not clear how this relates to statement **[24]**. Maybe statement **[24]** should be spread over two years to make more sense. (ACME) • There is no mention of fractions as quantities or parts of a whole, it is all about fractions as numbers. Young children need a lot more time developing the fraction concept before they use fractions as numbers; this is number sequences gone mad. To make the transition from partitioned fractions to quantity fractions, children need to develop a sense of the size of fractions as numeric quantities (Gould, P. in Wright et al 2012). Counting in fractions will not help children develop a sense of the size of fractions and can lead to very limiting beliefs as to what fractions are. (Consultation no. 99)
Year 2- Number- Fractions	**68**	Count in halves and quarters to ten.	• This should be taught later. (Consultation no. 63) • The expectation of counting in fraction is excellent. (Consultation no. 11) • This necessitates a conceptual understanding

			of fractions which will be mainly a visual understanding. This will be difficult when pupils are still becoming confident in counting to 100 and in steps of whole integers. (Consultation no. 43)
Year 2- Number- Fractions	**70**	Ensure pupils count in fractions up to 10, starting from any number and using the ½ and $^{2}/_{4}$ equivalence (e.g. 1 ¼, 1 ½, 1 ¾, 2). This reinforces the concept of fractions as numbers and they can add up to more than 1.	• This statement should include a mention to counting in decimals. (Workshop)
Year 2- Geometry and Measures – Properties of shape	**74**	Identify and describe the properties of 3-D shapes including the number of edges, vertices and faces	• We welcome the use of correct vocabulary, but it needs to be clear that this should be in practical contexts. Note that this is not consistent with **[77]** where it talks about number of sides and isn't explicit about that term only being applicable to 2-D shapes. (Consultation – no. 54) • It would be useful if the Notes and Guidance specified which shapes the children are expected to know. (Consultation no. 94)
Year 2- Geometry and Measures – Properties of shape	**75**	Identify 2-D shapes on the surface of 3-D shapes, for example rectangle and square on a cuboid, circle on a cylinder, triangle on a pyramid	• Please note that a cuboid doesn't necessarily have any square faces, as squares are rectangles: removing square would be accurate. (Workshop + Consultation no. 54)
Year 2- Geometry and Measures – Properties of shape	**78**	Ensure pupils read, write and accurately name 2-D and 3-D shapes and practise using a ruler to draw polygons accurately.	• Some six year olds may not have the fine motor skills to do this – it may be easier and more appropriate for them to use technology. (Workshop + Consultation no. 54) • You cannot draw polygons accurately with just

			a ruler; you have to have some kind of angle measuring device too. (Workshop + Consultation – no. 95 [MA and ATM joint primary group], no. 54) • It would be useful to give more details about the kind of polygons that should be drawn. (Consultation no. 39)
Year 2 – Geometry and Measures- Position, direction, motion	**79**	Use mathematical vocabulary to describe position, direction and movement, including rotation as a turn and in terms of right angles for quarter and half turns (clock-wise and anti-clockwise), and movement in a straight line.	• Right angles are conceptually demanding and are best left until later. (Workshop + Consultation no. 54)
Year 2 – Geometry and Measures- Position, direction, motion	**80**	Ensure pupils are taught the concept and vocabulary of angles by applying rotations, including in practical contexts; e.g. pupils themselves moving in turns, giving instructions to other pupils or programming robots	• Welcome reference to the use of technology, need to be careful that the robot works in quarter turns and doesn't require degrees for most children of this age. (Workshop + Consultation no. 54 and 03)
Year 2- Geometry and Measures- Measures	**81, 82, 83**	• choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm/mm); mass (kg/g); temperature (°C); volume and capacity (litres/ml) to the nearest appropriate unit using rulers, scales, thermometers and measuring vessels [81] • compare and order lengths, mass, volume/capacity and record the results using >, < and = [82] • read relevant scales to the nearest numbered unit [83] • compare, measure and record the following using standard units for: - lengths and heights (e.g. long/short, longer/shorter, tall/short,	• These statements ought to precede line **[30]** in meaning, rather than come a year later. They follow from line **[31]** about comparing quantities, including use of non-standard units**.** (Workshop) • The expectations are too high and the range of standard units (m/cm/mm) is inappropriate at **[81].** Young children would not be able to deal with the size of the conversion numbers;

		double/half) - lengths and heights (metres, centimetres) - mass (grams, kilograms) - capacity and volume (litres) - time (hours, minutes, seconds) [30]	they are too large. (Consultation no. 99)
Year 2- Geometry and Measures- Measures	**82**	Compare and order lengths, mass, volume/capacity and record the results using >, < and =	• Care needs to be taken on the use of these symbols – they must be used in respect of quantities and measures as opposed to objects. (Workshop)
Year 2- Geometry and Measures- Measures	**81, 85, 86, 87, 88**	• Choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm/mm); mass (kg/g); temperature (°C); volume and capacity (litres/ml) to the nearest appropriate unit using rulers, scales, thermometers and measuring vessels [81] • Recognise and use symbols for pounds (£) and pence (p); recognise coins and no.tes of different values; combine amounts to make a particular value and match different combinations of coins to equal the same amounts of money; add and subtract money of the same unit. [85] • Ensure pupils continue to practise using standard units of measurement to develop increasing accuracy. They should use the vocabulary and write associated symbols accurately. [86] • Ensure pupils regularly practise telling and writing the time. [87] • Ensure pupils regularly practise counting and recognising coins. They should accurately use the symbols '£' and 'p', including combinations, and say the amounts of money with confidence. Pupils should also regularly practise addition and subtraction of money in the same unit, including giving change. [88]	• Children should be continuing to compare and order in practical, meaningful contexts across the curriculum during KS1. They should read scales to the nearest numbered unit **[83]** where the numbers are consistent with their familiarity with number (e.g. cm on a ruler, grams or kilograms on a set of scales). (Workshop + Consultation no. 54)
Year 2-	**84**	Tell and write the time to 5 minutes including quarter past/to the hour and	• It would be better to teach this later. Drawing

Geometry and Measures- Measures		draw hands on a clock face to show these times	hands on a clock face leads to common misunderstandings where children don't appreciate that the hour hand is always moving e.g. at half past ten it is half way between ten and eleven. Children need lots of experience with real clocks and watches as suggested in **[87].** (Workshop + Consultation no. 11 and 54) • Read the analogue clock in one direction (clockwise) only in the early stages. The 5 past 7 and 7.05 link up. Do quarter past but NOT quarter to. (Consultation no. 11) • This statement should also be linked to fractions. (ACME) • Children are not conceptually ready to learn this. (Consultation no. 66)
Year 2- Geometry and Measures- Data	**89 - 90**	• Construct and interpret pictograms, tables and simple graphs [89] • Ensure pupils have regular practice in interpreting data diagrams so they become proficient in extracting information. They should apply their knowledge in science and other subjects as appropriate. [90]	• Constructing graphs is the middle of the data handling process. Before constructing graphs children should be discussing and identifying questions which can be answered with the use of data, and then collecting data. (Workshop + Consultation no. 54) • What is meant by 'construct'? Does this mean drawing by hand? Does it imply organizing data? (Workshop) • What is meant by 'interpret'? Does this mean simply reading off information, or interpreting the implications of the data in the context? (Workshop)

			• What is meant by 'simple graphs'? (Workshop) • Pictograms are used in the media, but are not a particularly useful, or easy, means of displaying data. (Workshop) • What is meant by 'data diagrams' in the N&G? **[90]** (Workshop) • Rather than applying knowledge in science, science provides a valuable context for learning data handling skills. (Workshop) • This is great opportunity for cross curricular links, particularly science, design and technology and geography. (Consultation no. 54)
Year 2- Geometry and Measures- Data	**89**	• Construct and interpret pictograms, tables and simple graphs [89]	• This is a good example of a broad statement as used in the Scotland curriculum. The details identifying the sort of processes that should be accessed in each year should be relegated to Notes and Guidance. (Consultation no. 76)
Year 3- Number- Number, place value and rounding	**94**	• Count in multiples of 2, 3, 4, 5, 8, 10, 50 and 100 from 0; give 10 or 100 more or less than a given number.	• It would be more accurate to say 'count multiples of…' or 'count in steps of ..' (Consultation no. 94)
Year 3, Number, Addition and	**98**	• Solve word problems including missing number problems, using number facts, place value, and more complex addition and subtraction.	• Suggest omitting 'word' as it limits the problem-type. (ACME)

Subtraction			• If this means solving 'multiple steps' problem, it should be indicated. (ACME)
Year 3, Number, Addition and Subtraction	**99**	• Ensure pupils continue to practise the use of column addition and subtraction with increasingly large numbers, using carrying for addition and borrowing for subtraction.	• The term borrowing in subtraction is not a mathematical term, nor does it convey a common understanding of a subtraction method. One could interpret this as 'decomposition' – exchanging tens for ones (units), however the term borrowing could also be interpreted as the equal addition method for subtraction, which is not always well understood and should be avoided. (Workshop and consultation no. 6, 46, 56 and Anonymous) • Language in a curriculum should avoid colloquial use that does not carry mathematical meaning. (ACME) • Column addition and subtraction should be taught later. (Consultation no. 63) • This statement should be less specific and could be replaced by *' solving increasingly difficult addition and subtraction problems using written methods*'. (Consultation no. 86)
Year 3 – Number – Multiplication and Division	**102**	• Write and calculate mathematical statements for multiplication and division within the multiplication tables; and for 2-digit numbers x 1-digit numbers, using mental and written methods	• Terminology of 'mathematical statements' is unclear. Are these recorded vertically, horizontally, as words, as symbols? (Consultation no. 49 and 68)
Year 3 – Number – Multiplication	**105**	• Ensure pupils develop efficient mental methods. For example, pupils should use commutativity (e.g. 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240) and multiplication and division facts (e.g. using 3 x 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3 to calculate 30 x 2 = 60, 60 ÷ 3 = 20 and 20 = 60 ÷ 3).	• The first bracketed statement involves associativity as well as commutativity.

and Division- No.tes and Guidance			(Workshop)
Year 3- Number, Fractions	**107**	• Identify, name and write unit fractions up to $^{1}/_{12}$	• Why limit knowledge to denominators of 12 or less? Students can name 1/24 or 1/100. They need to understand this to make the links with percentages. Introduce larger denominators earlier. Explicitly make the link with division. Only one part at first, then multiple parts later. This idea must be fully understood for pupils to convert fractions to decimals. (Consultation no. 11) • Sixths and sevenths are to be taught in Y3, but 6 and 7 times tables are to be taught in Y4. It would be better if they were taught alongside each other. (Consultation no. 53)
Year 3- Number, Fractions	**109**	• Recognise fractions which are equivalent to 1 and pairs of fractions that add up to 1	• The statements about complements are really well made and provide a good improvement of the curriculum. (Consultation no. 11)
Year 3- Number, Fractions	**110**	• Perform calculations with addition and subtraction of fractions with the same denominator within one whole (e.g. $^{5}/_{7} + {}^{1}/_{7} = {}^{6}/_{7}$) [110]	• It is more important to work on equivalence and locating fractions on a number line so children begin to appreciate that fractions are numbers. (Consultation no. 54) • In the PoS fractions should be written correctly with horizontal division lines. (Consultation no. 54)
Year 3 – Number -	**111**	• Count up and down in tenths; recognise that tenths arise in dividing an object into tenths and in dividing single digit numbers or quantities by	• Tautology: 'tenths arise in dividing an object into tenths'. It would be better to say '...*when*

Fractions		ten	*an object is divided into ten equal parts*'. (Workshop + consultation no. 54) • Anything can be divided by ten resulting in on tenth of it – the reference to single digit numbers or quantities is misleading. (Consultation- no. 54)
Year 3 – Geometry and Measures- Properties of shape	**117**	• Identify horizontal, vertical, perpendicular, parallel and curved lines	• These are different categories and should not have the same status. Suggest "*Distinguish between straight lines, straight line segments, and other lines; identify horizontal and vertical lines relative to the ground and on a page; identify pairs of lines that are perpendicular or parallel to each other*'. (ACME)
Year 3 – Geometry and Measures- Properties of shape	**118**	• Use a compass to draw circles and arcs with a given radius.	• This should say: 'pair of compasses'. (Workshop + Consultation no. 54) • Note that using a pair of compasses is difficult for some children – why prescribe for Y3 to do this, particularly with a given radius! (Consultation no. 38, 41 and 54) • Using a compass is mentioned in Y3 but not again – they will have forgotten about them by Y7. It would be better to introduce them later on instead/as well i.e. Y4, 5 and 6. (Consultation no. 38) • It would be better to focus on drawing other shapes accurately and knowing their properties. (Consultation no. 46)

Year 3 – Geometry and Measures- Properties of shape	**119**	• In Year 3, teachers should extend pupils' knowledge of the properties of shapes, using more precise mathematical vocabulary including polygon, non-polygon and polyhedron.	• Imprecise vocabulary: Does it mean that pupils should distinguish between regular and irregular polygons; symmetrical and non-symmetrical polygons; or does it mean they should be able to name polygons according to the number of sides? Does it mean they should be able to define a polygon? The word 'non-polygon' is not much used in mathematics. (Workshop)
Year 3 – Geometry and Measures- Properties of shape	**120**	• Ensure pupils extend their use of the properties of shapes. They should be able to describe the properties of 2-D and 3-D shapes using accurate vocabulary, including acute and obtuse angles, turns and lines.	• Beginning to recognise angles that are less than or greater than a right angle is one thing, using 'acute' and 'obtuse' is quite another. Why must this be done in Y3? (Consultation no. 54)
Year 3 – Geometry and Measures- Properties of shape	**121 and 173** **(also 235, 291 and 296)**	• Ensure pupils practise measuring and drawing straight lines in centimetres and millimetres, and circles of different sizes with a given radius using a compass. Ensure they also understand the terms horizontal and vertical lines. [121] • Ensure pupils continue to classify shapes, extending to classifying different rectangles and triangles. Ensure pupils continue to practise drawing circles with a compass and use the related vocabulary.[173]	• This could also be done programmatically rather than with a pair of compasses. (Consultation no. 03 and 56)
Year 3- Geometry and Measures- Measures	**122-123-130**	• Recognise and use full names and abbreviations for metric units of measure [122] • Measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml); and time (hours/minutes/seconds) [123] • Ensure pupils continue to practise measuring using the appropriate tools and units of measure. They should progress to using a wider range of measures, including comparing and using mixed units accurately (e.g.1 kg and 200g) and simple comparisons of mixed units	• The expectations are unrealistic for Y3, and would be challenging for many in Y6 and Y7. Equivalent units require children to have a secure grasp of place value up to and beyond 1000 and thousandths; this is unlikely for most children in Y3. (Consultation no. 54) • Note also that this knowledge is assumed in

		(e.g. 5m = 500cm). [130]	earlier years, even in year 1. (ACME)
Year 3- Geometry and Measures- Measures	**124**	• Measure the perimeter of simple 2-D shapes	• Welcome the separation of perimeter from area but it would be worth emphasising that perimeter needs to be understood as a length. (Consultation no. 54)
Year 3- Geometry and Measures- Measures	**125**	• Tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12 hour and 24 hour digital clocks	• This is remarkably prescriptive. The crucial thing is that children have lots of experience with real clocks (not drawing times on clock faces **[84]** or using clocks that don't have geared hands). (Consultation no. 54)
Year 3- Geometry and Measures- Measures	**130**	• Ensure pupils continue to practise measuring using the appropriate tools and units of measure. They should progress to using a wider range of measures, including comparing and using mixed units accurately (e.g.1 kg and 200g) and simple comparisons of mixed units (e.g. 5m = 500cm).	• This should be written '5 m = 500 cm', with a space between the numeral and the unit. (Workshop)
Year 3- Geometry and Measures- Measures	**131**	• Ensure pupils use both analogue and digital clocks throughout the day so that they become fluent in telling the time.	• Linking ¼ to 7 and 6.45 is difficult. Leave 'minutes to' until later. (Consultation no. 11)
Year 3- Geometry and Measures- Measures	**132**	• Ensure pupils continue to practise recognising the value of coins, addition and subtraction of amounts, including compound units, and giving change using manageable amounts.	• "Compound units" usually means units such as 'miles per hour' or 'mass per unit volume'. These require ratio and/or division which has not yet been introduced so probably that is not what is meant. It probably means mixed units such as 'three pounds and ten pence' or 'five metres and thirty centimetres'. If it does mean compound units this has to wait until children experience use of these in a variety of contexts and have some idea of what 'rate'

			means. (Workshop + Consultation no. 54)
Year 3- Geometry and Measures- Data	**133, 134, 135**	• Read, interpret and present data using pictograms and bar charts with scales • Solve problems using information presented in pictograms, bar charts and tables. • Ensure pupils use both horizontal and vertical representations as well as scales for pictograms, for example, where each picture represents 10 bags.	• The PoS now refers to bar charts, which were not mentioned explicitly in Y2. (Workshop) • What is the meaning of 'present' here? How does this relate to 'construct' in Y2? **[89].** (Workshop) • What is the difference between 'read' and 'interpret'? Why is 'read' included here but not in Y2? (Workshop) • Again, posing problems and collecting data need to precede using graphs to display it. (Workshop) • There could be a link here to using the scales on measuring instruments. (Workshop)
Year 4- Number – Number, place value and rounding	**136**	• Read and write numbers to at least 10,000	• This is welcomed. (Consultation no. 63)
Year 4- Number – Number, place value and rounding	**139**	• Count in multiples of 2, 3, 4, 5, 6, 7, 8, 9, 10, 25, 50, 100 and 1000 from any given number, and 10 or 100 more or less than a given number [139] • Count in multiples of 2, 3, 4, 5, 8, 10, 50 and 100 from 0; give 10 or 100 more or less than a given number.[94]	• This can be confusing. The statement would benefit from adding the word 'give', after the second '10', as it appears in statement **[94].** (Workshop) • This probably means 'count multiples of ..' or 'count in steps of ..' (Consultation no. 54) • We are puzzled about the reasons for

			counting in all these multiples as some are more useful than others. (ACME)
Year 4- Number- Number, place value and rounding	**141**	• Read and write negative numbers; order, count forwards and backwards with positive and negative whole numbers through zero	• Negative numbers are conceptually very challenging. They need to be introduced in meaningful practical contexts such as temperature, below sea level, debt etc. as well as a consequence of subtraction. (Consultation no. 54) • Negative numbers are a difficult concept for Year 4. (Consultation no. 82)
Year 4 – Number - Number, place value and rounding	**142 and 147**	• Read Roman numerals to 100 and understand how Hindu-Arabic numerals included the concept of zero and place value. [142] • Roman numerals should be put in their historical context so pupils understand that there were different ways to write whole number and that Hindu-Arabic numerals introduced the important concept of zero and place value.[147]	• This reference to mathematics history is welcomed. However, it is partly inaccurate and should be corrected. The earliest known place value system didn't use base 10 (Babylonian and Mayan) and survived for thousands of years without a symbol for zero. The origin of zero is not conclusively known. There were other number systems before the Romans and focusing solely on Roman numerals might be perceived as Eurocentric. (Consultation – no. 54 – no. 95 [ATM and MA joint primary group]) • This might be better addressed as a key stage item to explore and compare number systems with the teacher judging the level of challenge as appropriate. (Consultation no. 39) • The learning aims of this statement should be explained. (Consultation no. 80)

Year 4- Number- Number, place value and rounding	**145**	Ensure pupils read and write 4-digit numbers accurately, including the use of zero as a place holder.	• 'Including zero as a placeholder': this is also relevant for numbers up to 100 so should have been put in KS1 as well, or in KS1 it should say 'excluding zero as a placeholder' (which does not make sense). (Workshop)
Year 4- Number- Number, place value and rounding	**146**	Ensure pupils are applying their mathematics, including completing number sequences and finding the difference.	• It might be better to say: '*Ensure pupils are applying their mathematics, including describing, extending and explaining number sequences and patterns, by, for example, finding the difference between terms'*. 'Finding the difference' could be interpreted in a subtraction context. (Workshop)
Year 4- Number- Addition and Subtraction	**148**	Add and subtract numbers using formal written methods with up to 4 digits	• This requires children to add and subtract numbers up to 4 digits – but the corresponding Notes and Guidance **[151]** shows examples with 5 digits. (Workshop) • There are concerns over the prescription of method. (Consultation- no. 54) • What is a formal written method? Is it the same as or different from column methods? (Consultation no. 49 and 68)
Year 4- Number- Addition and Subtraction	**149**	Accurately add and subtract numbers mentally including two 2-digit numbers	• This is already mentioned in statement **[50]** – no progression. (Anonymous)
Year 4- Number- Addition and	**150**	Estimate, within a range, the answer to a calculation and use inverse operations to check answers.	• This is welcomed. (Consultation - no. 54)

Subtraction			
Year 4- Number- Addition and Subtraction – No.tes and Guidance	**151 (and 136, 138)**	Ensure pupils continue practising formal written methods and mental methods with increasingly large numbers, and include the terms 'sum' and 'difference'. For mental calculations, include increasingly large numbers, for example, 12,462 - 2,400 = 10,062 or 12,462 + 600 = 13,062.	• The official *Systeme International* publication argues that use of the comma is to be avoided as several countries use this symbol as the decimal point, and recommends that a space be left after every three digits working from right to left, except in the case of 4-digit numbers such as 2400. (Workshop + Consultation no. 54 and 94).
Year 4- Number- Addition and Subtraction	**152**	Ensure pupils say and write the numbers correctly and with precision, so that they are clear about place value and confident when working with mental calculations. This will prepare them for Year 5, when pupils are taught to calculate the sum and difference of two decimal numbers (up to 2 decimal places).	• In principle, signposting is a good idea but it should be realistic. Why will children be able to calculate with numbers up to 2 decimal place when they've not even worked with 1 decimal place? (Consultation no. 54) • This statement assumes a year by year curriculum – it could be reworded to refer to progression. (ACME)
Year 4 – Number – Multiplication and Division	**153**	Recall multiplication and division facts for multiplication tables up to 12 x 12	• I welcome the inclusion of learning tables in Year 4. (Consultation no. 2 and 63) • Whilst it is desirable for children to know multiplication facts, this is an unrealistic expectation for most children at this age. Rote learning of multiplication facts doesn't encourage teachers to support children's understanding of number relationships so they can quickly figure out those multiplication facts that they cannot quickly recall using those that they do remember. For example it is easy to multiply by ten or two and provided you are good at addition and subtraction it is relatively easy to derive all other multiplication facts.

			Note that in USA where children are expected to 'know' multiplication facts up to 12×12, imperial measures are still used. (Consultation- no. 4, 14, 56 and 54) • It would make more sense to aim for *instant* recall up to 9x9 in Year 4 and to extend this, if possible, to 12x12 by Year 6 (given that only targets for the end of Year 6 are to be statutory. (Workshop)
Year 4- Number – Multiplication and division	**155**	Multiply or divide 2-digit and 3-digit numbers by a 1-digit number using formal written methods; interpret remainders appropriately as integers	• The word 'whole number' would be more appropriate here than 'integer'. Given that the remainder will always be a positive number, and that integers include the negatives, this seems an inappropriate time to introduce the word. (Workshop + Consultation no. 54) • The word 'interpret' does not have a clear meaning in this context. It might be better to say '…. written methods; *leaving remainders as whole numbers'.* (Workshop) • This is inappropriate for children at this age. There is no evidence that children can cope with formal division methods by age 11 (see for example Ofsted's *20 Successful Primary Schools*.) It is more important that children use their developing understanding of multiplicative relationships to solve practical problems that require both multiplication and division. Remainders need to be introduced with care. Division is a notoriously difficult operation to understand, particularly when remainders arise. (Consultation no. 54)

			• What is a formal written method? Is it the same as or different from column methods? (Consultation no. 49 and 68)
Year 4- Number- Multiplication and Division- No.tes and Guidance	**159**	Ensure pupils continue to practise mental methods and extend this to 3-digit numbers to derive facts, for example 300 x 2 = 600 into 600 ÷ 3 = 200. Pupils should also use the distributive law to derive facts, for example, 30 x 7 + 9 x 7= 39 x 7.	• The 'derived fact' should be 600 ÷ 2 = 300. (Anonymous) • The final example is the wrong way round: if you are given 30 x 7 + 9 x 7, you don't need to know that it's 39 x 7, you just work it out, whereas it is important to know that 39 x 7=30 x 7 + 9 x 7 in order to facilitate mental or written multiplication. (Workshop) • It is actually more efficient to find 39x7 is (40x7) – (1x7). (Consultation no. 97)
Year 4- Number – Fractions and Decimals	**160, 165, 166, 169, 179**	• Identify and name equivalent fractions of a given fraction with denominator not greater than 12 [160] • compare numbers with the same number of decimal places up to 2 decimal places [165] • find the effect of dividing a 2-digit number by 10 and 100, identifying the value of the digits in the answer as units, tenths and hundredths [166] • Ensure pupils' understanding of decimal place value is extended to tenths and then hundredths. This will prepare them for Year 5 when they are taught how to relate the decimal notation to division of 2-digit numbers by 10 and later 100, and to the groups of fractions for $^{1}/_{10}$ and later $^{1}/_{100}$. [169] • convert between different units of measure, for example: kilometre to metre; metre to centimetre; centimetre to millimetre; kilogram to gram;	• Line **[165]** needs hundredths, which only appear in line **[166]** and not under the 'fractions' heading. In fact line **[160]** expressly excludes hundredths, taken at face value. Line **[169]** says that hundredths are going to come in year 5, but line **[165]** requires them in year 4. See also line **[179].** (Workshop) • Year 4 are expected to understand decimals to 2 decimal place but also to be able to convert g to Kg and m to Km. To do this, they need an understanding of decimals to 3 decimal place. (Consultation no. 4) • Statement **[169]** assumes a year by year curriculum – it could be reworded to refer to progression. (ACME)

		litre to millilitre; hour to minute; minute to second; year to month; week to day [179]	
Year 4- Number- Fractions- No.tes and Guidance	**164**	Ensure pupils continue practising to add and subtract like fractions within one whole and extend this to equivalent fractions. Ensure pupils practise counting as often as possible using simple fractions and decimal fractions both forwards and backwards.	• It would be more informative to say: '*Ensure pupils continue practising to add and subtract like fractions i.e. those with the same denominator*'. (Workshop) • Children need a thorough understanding of equivalence and fractions as numbers before they start calculating with fractions. (Consultation – no. 54)
Year 4- Number- Decimals	**165 - 167**	• Compare numbers with the same number of decimal places up to 2 decimal places [165] • Find the effect of dividing a 2-digit number by 10 and 100, identifying the value of the digits in the answer as units, tenths and hundredths [166] • Recognise and write decimal equivalents to ¼, ½, ¾ and any number of tenths and hundredths [167]	• Very ambitious introduction to decimals – most students will need far more time to work on decimal- fraction equivalence. Technology e.g. use of a calculator would be extremely helpful in supporting children's understanding. (Consultation no. 54)
Year 4- Geometry and Measures- Properties of Shape	**171**	• Compare and classify geometric shapes, including squares, rectangles and triangles based on their properties and sizes	• Square are rectangles. (Workshop)
Year 4- Geometry and Measures- Properties of	**172**	• Identify acute and obtuse angles and compare the size of different angles.	• This should be taught in year 6. (Anonymous)

Shape			
Year 4- Geometry and measures- Position, direction, motion	**173**	• Ensure pupils continue to classify shapes, extending to classifying different rectangles and triangles. Ensure pupils continue to practise drawing circles with a compass and use the related vocabulary.	• Classifying rectangles can only be done according to side length ratios so should wait until year 6 or later. Does it mean 'classifying quadrilaterals'? (Workshop) • This statement should say: 'a pair of compasses' rather than 'a compass'. (Workshop)
Year 4- Geometry and measures- Position, direction, motion	**174, 175, 177**	• Describe positions, and movements between positions, on a 2-D grid, and as coordinates in the first quadrant [174] • Plot specified points and draw sides to complete a given polygon [175] • Ensure pupils draw a pair of labelled axes in one quadrant and regularly read, write and use pairs of coordinates, e.g. (2, 5). [176]	• Drawing axes and plotting coordinates are challenging for many in Y7, Y4 is too soon. Using contexts that require children to locate points on a grid e.g. a treasure map or street map is appropriate at this stage. Also using technology to generate graphs is appropriate. (Workshop + Consultation no. 16 and 54)
Year 4- Geometry and Measures- Measures	**179**	• Convert between different units of measure, for example: kilometre to metre; metre to centimetre; centimetre to millimetre; kilogram to gram; litre to millilitre; hour to minute; minute to second; year to month; week to day.	• There is no. evidence that Y4 children will be able to do this. They need place value knowledge of thousands and thousandths, which they won't have met yet. Mm and cm, days and weeks may be appropriate. (Workshop + Consultation no. 54) • Note that there are often mismatches between measure and decimal understanding in this current version of the curriculum. (ACME)
Year 4- Geometry and	**180-181**	• Measure and calculate the perimeter of a rectilinear figure, where each side is labelled in centimetres and metres	• Research shows that area and perimeter should be taught separately as children often

Measures- Measures		• Find the area of squares and rectangles and related composite shapes	muddle the concepts. (Consultation no. 82)
Year 4 – Geometry and Measures- Measures	**181**	• find the area of squares and rectangles and related composite shapes	• Square are rectangles. Area is a challenging concept and at this stage children should just be working on rectangles. Note that 'composite shapes' is repeated in **[239]** where it would be more appropriate. (Workshop + Consultation no. 54) • Bryant (2009: 18) confirms that area is a challenging concept for children. (Difficulties with area, including potential for confusion with perimeter are mentioned at several points in this report). (Workshop)
Year 4 – Geometry and Measures- Measures	**182**	• Read and convert time between analogue and digital 12- and 24- hour clocks	• Yes for 12 hour clock but leave 24 hour clock until Y6. (Consultation no. 11)
Year 4 – Geometry and Measures- Measures	**185**	• Ensure pupils are introduced to area, initially by counting squares (e.g. cm^2 squares) and later using perimeter measurements to calculate areas.	• Probably 'side lengths' are meant instead of perimeter. (Workshop + Consultation no. 54)
Year 4- Geometry and Measures- Data	**186, 187, 188**	Read, interpret and solve problems using information in bar graphs, including reading scales on the axes. [186] Ensure pupils continue to practise interpreting a variety of bar graphs so that they can read, write, analyse and solve problems confidently in Year 4. They should continue to apply their knowledge in science and other	• The PoS shows no clear progression from Y3, except the reference to 'bar graphs' which are not clearly distinct from 'bar charts'. (Workshop) • Both horizontal and vertical representations require putting variables on both axes, so the example in the N&G, specifying which variable

		subjects as appropriate [187]. Ensure pupils use horizontal and vertical representations of bar graphs so that pupils are confidently able to interpret and write the variable on the horizontal axis (e.g. shoe size) and the frequency on the vertical axis (e.g. number of people).	goes on which axis is unnecessarily confusing. (Workshop) • There should be a link here to the use of co-ordinates in geometry, which also uses continuous scales on both axes. (Workshop)
Year 5-Number-Number, place value, approximation and estimation	**193**	Read Roman numerals to 1000 (M) and recognise years written in Roman numerals.	• The aim of this learning objective should be explained. It does not seem to be an essential in the curriculum. (Consultation no. 40, 46, 56, 63 and 94)
Year 5-Number-Addition and Subtraction	**195**	Add and subtract whole numbers with up to 5 digits, including using formal written methods	• What is a formal written method? Is it the same as or different from column methods? (Consultation no. 49 and 68) • Adding and subtracting whole numbers up to 5 digits is no harder than adding and subtracting 3 digit numbers - there is a lack of progression. (Consultation no. 56)
Year 5-Number-Addition and Subtraction	**198**	Ensure pupils continue to practise fast responses for mental calculations with increasingly large numbers, for example: 12,462 – 2,300 = 10,162.	• Mental calculations can include jotting. This should be made clear. (Consultation no. 94)
Year 5 – Number – Multiplication and division	**202**	Multiply numbers up to 4-digits by a 1 or 2-digit number using a formal written method, including long multiplication	• What is a formal written method? Is it the same as or different from column methods? (Consultation no. 49 and 68) • Potentially some ambiguity of terminology: Long multiplication usually suggests using a

			written strategy to multiply by a two-digit number or greater rather. Hence short division being multiplication by a single digit. Similarly for short and long division. (Consultation no. 49, 46 and 68)
Year 5 – Number – Multiplication and division	**204**	Divide numbers up to 4 digits by a 1-digit number and 10 and interpret remainders appropriately	• There is no indication if this should be done through a written method or not. (Consultation no. 49 and 69)
Year 5 – Number – Multiplication and Division – No.tes and Guidance	**209**	Ensure pupils record answers for non-integer division in different ways, including: with remainders, fractions, decimals or with rounding; for example, 98 ÷ 4 = 24 r 2 = 24½ = 24.5 ≈ 25.	• 'Non-integer division' is a computer programming concept. Taken literally, it has to mean 'division by a non-integer', i.e. by a vulgar or decimal fraction, but here the division example involves two integers. Dropping 'non-integer' would maintain the meaning of the sentence. (Workshop) • It is mathematically incorrect to put 25 at the end. The sentence should stop at 24.5 and be followed by something like 'Rounded up, this answer would be 25'. (Workshop + consultation no. 81)
Year 5 – Number- Fractions	**210**	Compare and order fractions with different denominators	• Pupils cannot add and subtract fractions with different denominators until they fully understand equivalences. This is why so many adults learn the procedure by rote and then immediately forget the 'trick'. Leave until Year 6. In this area as in other through the curriculum: slow down. It will pay off later. (Consultation no. 11)

Year 5 – Number- Fractions	**212 (and 260)**	Add and subtract fractions with the same denominator and related fractions;, write mathematical statements that exceed 1 as a mixed number: (e.g. $^{2}/_{5} + {}^{4}/_{5} = {}^{6}/_{5} = 1{}^{1}/_{5}$)	• I welcome the introduction of some fractions in upper key stage 2. (Consultation no. 2)
Year 5- Number- Decimals	**217**	Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents	• Children should be able to do more than just recognise thousandths and relate them to tenths, hundredths and decimal equivalent. The word '*recognise*' should be replaced by an action. (Workshop) • There should be a more coherent linking between fractions and decimals, not just tenths and hundredths (e.g. 4x25 = 100 links to ¼ = 0.25). (Workshop)
Year 5 and Year 6 – Number- Decimals	**217 and 267**	• Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents [217] • identify the value of each digit to three decimal places and multiply and divide numbers up to three decimal place by 10, 100 and 1000 [267]	• These understandings would have to be in place for children to do some of the measures conversions described in earlier years for measures. (Workshop)
Year 5- Number- Decimals	**221**	• Ensure pupils recognise and use complements of 1 using addition and subtraction facts and place value, e.g. 0.83 + 0.17 = 1.	• Welcomes the use of the word 'complements' to promote connections between mathematical ideas. (Consultation no. 11)
Year 5- Number- Decimals	**222**	• Ensure pupils continue to practise counting forwards and backwards using decimal fractions, and mental addition and subtraction of tenths and 1 digit whole numbers and tenths.	• Typo: 1-digit. (Workshop)
Year 5- Number-	**224 and 225**	• Recognise the per cent symbol (%) and understand that per cent relates to "number of parts per hundred" for example that 100% represents a whole quantity and 1% is $^{1}/_{100}$, 50% is $^{50}/_{100}$, 25% is $^{25}/_{100}$,	• Good clarification and information on equivalent fractions that should be taught. Though percentages and decimals should be

Percentages		etc. [224] • Write simple fractions as percentages and decimals as percentages (e.g. ½ = 50% = 0.5).[225]	linked as well. (Anonymous)
Year 5- Number- Percentages	**226**	• Ensure pupils make connections between percentages, fractions and decimals. They should recognise that percentages are operators	• To help teachers, this statement needs to make it clear that 'Ensure pupils make connections...' means more than 0.5=1/2=50%. It needs to be made explicit that ½ = 1 ÷ 2 = 0.5 which is the same as 0.50 which represents 50/100 or 50%. (Consultation no. 86)
Year 5- Geometry and Measures- Properties of shapes	**229**	• Recognise and compare different triangles including: isosceles, equilateral and right-angled; identify and name the following: parallelogram; rhombus; trapezium	• This reinforces a common misconception about triangles – triangles can be named according to side lengths: equilateral, isosceles and scalene and according to the size of the greatest angle: acute, right and obtuse-angled. All triangles require a side and angle name e.g. scalene right-angled or acute-angled isosceles, in special cases e.g. equilateral we may not bother with the angle part. (Workshop + Consultation no. 54) • This appears to be about the classification of shapes described in line **[173]** earlier. (workshop)
Year 5 – Geometry and Measures – Properties of	**230**	• Construct shapes from given dimensions; state and use properties of a square and rectangle	• This might imply the use of pairs of compasses to construct shapes. It would be better to include the use of technology and practical materials. (Workshop + Consultation

shapes			no. 54) • 'Construct' usually means 'using pair of compasses and straight edge', does this mean 'draw'? (Workshop)
Year 5- Geometry and Measures - Properties of shapes	**233**	• Include the term 'diagonal' and related properties of diagonal with reference to angles and sides.	• 'Properties of diagonals' is mysterious. Probably means '*properties of diagonals of quadrilaterals*' because triangles do not have diagonals. What is meant by 'with reference to angles'? Does it mean 'whether they are bisected or not' or is it referring to Euclidean theorems (too early)? (Workshop and ACME)
Year 5 – Geometry and Measures – Position, direction, motion	**234**	• Identify, describe and represent the position of a shape following a reflection or translation using the appropriate vocabulary.	• Transformations (i.e. operations on shapes) are best left to the KS3 PoS given their conceptual challenge. This could be replaced by: '*Children should create designs and patterns that involve repeating shapes'*. (Workshop + Consultation no. 54) • Bryant (2009: 34) reports on the difficulties children, including many in secondary schools, experience with the transformation of shapes. (Workshop)
Year 5- Geometry and Measures- Measures	**236**	• Add, subtract, multiply and divide units of measure (e.g. length, mass, volume, money) using decimal notation	• The Notes and Guidance should include some context details. The use of measures needs to be practical and meaningful. (Consultation no. 54)
Year 5- Geometry and Measures-	**237**	• Understand and use basic equivalencies between metric and common imperial units and express them in approximate terms	• Repeats conversion, which has already appeared in lower KS2 but that was possibly too early, given the need for full understanding

Measures			of operations and systems required. (Workshop + consultation no. 54) • The progression here is confusing; children need to understand equivalence of metric units fully, in conjunction with the decimal system of numbers. Approximate relations with imperial units are a separate issue which arises through everyday experiences where we still use imperial units. (Workshop)
Year 5- Geometry and Measures- Measures	**238**	• Measure force in Newtons (N)	• This is an extremely challenging concept best left to KS3. (Workshop + Consultation no. 54) • The aim of teaching force in mathematics needs to be explained (Consultation no. 56)
Year 5- Geometry and Measures- Measures	**239**	• Calculate, estimate and compare the area of squares, rectangles and related composite shapes using standard units, including centimetre squared (cm2) and metre squared (m2)	• Cm^2 should be read as 'square centimetres'. (Workshop)
Year 5- Geometry and Measures- Measures	**240**	• Recognise volume in practical contexts, for example using sand and water, 1 cm^3 blocks or interlocking cubes to build cubes and cuboids.	• Great to see these practical approaches; sand and water should be used in KS1 when capacity is introduced. (Consultation no. 53, 54 and 82)
Year 5- Geometry and Measures- Measures	**241**	• Ensure pupils' calculation of area is extended to include scale drawings in metres (m and m^2) but without converting between cm^2 and m^2. Also ensure pupils' calculation of perimeter is extended to composite shapes.	• There has been no mention of scaling hitherto which is an essential aspect of developing multiplicative understanding. Scaling should start with lengths before moving on to area. (Workshop + Consultation no. 54) • Scaling area is usually met in KS4, so is not

			appropriate for Y5. There is substantial research to show the major difficulties children have in making this jump in early secondary (University of Leuven). (Workshop + Consultation no. 54)
Year 5- Geometry and Measures- Data	**242,243**	• Complete tables and bar graphs from given information and solve problems using data presented in bar graphs, tables and simple pie charts. • Ensure pupils regularly practise reading and interpreting so that pupils are confident in completing tables and bar graphs, and using data diagrams such as tally charts. Also ensure that pupils relate pie charts to angles and percentages.	• Tally charts can be used in year one. (Workshop) • Apart from the inclusion of pie charts, there is no obvious progression here from the PoS for Y2. It is not clear how 'complete' differs from 'construct', or indeed why one is more advanced than the other. (Workshop) • The introduction of tally charts – which are generally used in data collection - here is extraordinary, as there is still no reference to children collecting data. (Workshop) • There is no mention here of the importance of making decision about which kind of graph is appropriate for a particular context. (Workshop)
Year 6 – Number- Number, place value and rounding	**246 and 247**	• Recognise binary numerals to 15 (1111) and convert between binary and decimal numerals.[246] • Ensure pupils regularly practise saying, reading and writing numbers accurately. Binary numerals should be introduced so pupils are familiar with the concept of place value using a different base. [247]	• The concept of place value could be learned through the history of mathematics. (Consultation – no. 54 - no. 95 [ATM and MA Joint Primary Group]) • Restricting binary to 15 is insufficient to help children's understanding of place value. (Consultation – no. 54 - no. 95 [ATM and MA

			Joint Primary Group]). • Children would benefit more from becoming familiar with powers which, apart from squares and cubes, are not met until KS3. (Consultation - no. 54). • Exploration of different bases would be more helpful here, but base 2 would be interesting for the more able. (Anonymous) • This is well beyond an essential core. The learning aims should be explained. (Consultation no. 2, 56, 78 and 94)
Year 6- Number- Addition, subtraction, multiplication and division	**249**	• Multiply numbers with at least 4-digits by a 2-digit whole number using long multiplication	• This sentence could lead to confusion- it would benefit from a re-wording. (Workshop) • Potentially some ambiguity of terminology: Long multiplication usually suggests using a written strategy to multiply by a two-digit number or greater rather. Hence short multiplication being multiplication by a single digit. Similarly for short and long division. (Consultation no. 49 and 68) • Note that some authors define 'long multiplication' to mean, multiply multi-digit numbers, so the last three words here are redundant. Otherwise it should say 'a long multiplication algorithm' if that is what is meant. (ACME)

Year 6- Number- Addition, subtraction, multiplication and division	**250**	• Divide numbers up to 4-digits by a 2-digit whole number using long division, and interpret remainders as whole number remainders, fractions, decimals or by rounding	• Potentially some ambiguity of terminology: Long multiplication usually suggests using a written strategy to multiply by a two-digit number or greater rather. Hence short multiplication being multiplication by a single digit. Similarly for short and long division. (Consultation no. 49 and 68) • This should be taught later. (Consultation no. 63)
Year 6- Number- Addition, subtraction, multiplication and division	**252**	• Use estimation to check answers to calculations and determine, in the context of a problem, whether an answer should be rounded or written as a fraction or a decimal	• This should be ongoing for all year groups, not reserved for year 6. (Anonymous)
Year 6- Number- Addition, subtraction, multiplication and division	**253**	• Carry out combined operations involving the four operations accurately and state the order of operations	• It would be better to 'understand and use' rather than 'state the order'. (Consultation no. 54)
Year 6- Number- Addition, subtraction, multiplication and division	**259**	• For the order of operations include the use of brackets; for example, 2 + 1 x 3 = 5 and (2 + 1) x 3 = 9.	• The PoS should use a multiply symbol ×, rather than x for multiplication. (Consultation no. 54) • This could be introduced earlier and would be helpful in introducing symbolic reasoning. (Consultation no. 73)

Year 6-Number-Fractions	**260-261-262**	• Add and subtract mixed numbers and fractions with different denominators [260] • multiply simple unit fractions by fractions and pairs of proper fractions, writing the answer in its simplest form [261] • divide proper fractions by whole numbers [262]	• These are examples of statements that should be removed as there is too much risk of it being taught hurriedly. (Workshop) • Once tables are robust, one can work to establish the notion of equivalent fractions. At this point, fractions should have been routinely ordered and included on the number line via equivalent fractions; this is a prerequisite for fraction arithmetic and the notion of common denominators. You can then go on to robust addition and subtraction with like denominators, and of obviously related denominators (via tables) such as ½ +/- ¼ or 1/3 +/- 1/6 or ¾ +/ -3/8. (Workshop)
Year 6-Number-Fractions	**261**	Multiply simple unit fractions by fractions and pairs of proper fractions, writing the answer in its simplest form	• This might be interpreted as multiplying a unit fraction by a pair of proper fractions. As unit fractions are proper fractions, this could be replaced by '*multiply simple pairs of proper fractions, writing the answer in its simplest form*'. (Workshop) • This is an area of the curriculum that children find difficult and time would be better invested in ensuring that children understand the key concepts. (Consultation no. 56)
Year 6-Number-Fractions	**261-263**	Pupils should be taught to:	• **Extra statement to be added**: Once tables are robust, one can work to establish the notion of equivalent fractions (which

			will already be familiar in special cases). This is a prerequisite for fraction arithmetic, and the notion of common denominators. (Workshop)
Year 6- Number- Fractions	**262**	Divide proper fractions by whole numbers	• Students should also consider scenarios which give rise to division involving fractions as opposed to just learning the mechanics of it. (Consultation no. 16)
Year 6- Number- Fractions	**263**	• Associate a fraction with division to calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. $^{3}/_{8}$).	• The link between division and fractions should mentioned earlier, as it is fundamental to understanding fractions. (Consultation no. 11)
Year 6 – Number- Fractions- No.tes and guidance	**264**	Ensure pupils should use their understanding of the relationship between unit fractions and division to work backwards by multiplying a quantity that represents a unit fraction to find the whole quantity. For example, if ¼ of a length is 36cm then the whole length is 36 x 4 = 144cm	• This sentence could be made clearer, for example: '*Ensure pupils understand that, for example, if ¼ of a length is 36 cm then the whole length is 36 x 4 = 144 cm*.' (ACME)
Year 6- Number- Fractions	**266**	Pupils can use a calculator for a division calculation to convert a simple fraction to a decimal fraction, e.g. 3 ÷ 8 = 0.375. For simple fractions with infinite decimal equivalents, pupils should round the decimal to three decimal places.	• Rounding to 3d.p for a recurring decimal is very likely to incur error, children need to understand that some decimal equivalents terminate and others recur and be able to make appropriate choices about which representation to use in different contexts, including when to approximate with rounding e.g. $\frac{1}{3} = 1 \div 3 = 0.\dot{3} = 0.33 \; to \; 2d.p.$ (Consultation no. 54)

Year 6-Number-Decimals	**269**	Ensure pupils multiply decimals by whole numbers starting with the simplest cases, such as 0.4 x 2 = 0.8, and practical contexts, such as measures and money.	• An example of atomisation and the risk of losing the whole picture. There should be a note to indicate the link to multiplication **[248-254].** (Workshop)
Year 6-Number-Decimals	**270**	Ensure pupils are introduced to division of decimal numbers initially in practical contexts involving measures and money and by single digit whole numbers. They should recognise division calculations as the inverse of multiplication.	• The relationship between division and multiplication when the division is not exact is conceptually challenging. If remainders have been used the inverse of division may involve both multiplication and addition. Children need to understand that when division isn't exact there might be a remainder (as in sharing quantities and some are 'left over') or there might be a fraction of a group or share. In practical contexts students need to know when to round up or down. For example, if they calculate 3.2 buses for a school trip, they will need to order 4 buses; on the other hand when packing 55 apples in packs of 4, they need to interpret 13.75 as 13 full packs of apples. (Consultation no. 54)
Year 6 – Numbers Percentages	**273**	Recall and use equivalences between fractions, decimals and percentages.	• It would be better to use '*conversions between simple fractions, decimals and percentages'* and keep the word 'equivalent' here to mean 'equivalent fractions'. (Workshop)
Year 6 – Numbers Percentages	**274**	Ensure pupils understand that calculating a percentage of a quantity is the same as calculating a fraction of a quantity.	• The process of calculating is not 'the same'. A better formulation would be '*understand that a percentage of a quantity is a fraction of a quantity, as preparation for proportionality*'.

			(Workshop)
Year 6 – Numbers – Ratio and proportion	**275-279**	• Use ratios to show the relative sizes of two quantities [275] • Recognise equivalent ratios and reduce a given ratio to its lowest terms [276] • Recognise and use division in the context of fractions, percentages and ratio [277] • • Ensure pupils are introduced to and use the ratio notation and symbol (a:b) in the context of comparing quantities, sizes and scale drawings. [278] • Ensure pupils practise solving a wide variety of problems so that pupils are taught to apply ratio and proportion flexibly.[279]	• Recommend that any formal approach is left to KS3, while the use of scale drawings, adapting recipes etc. allows children to develop and apply proportional reasoning. (Consultation no. 54). • There is no preparation before this for the ideas of ratio and proportion e.g. there is nothing about the different meanings of multiplication and in particular the idea of scaling. These statements appear to be a summary of a large body of knowledge that all students are still developing way into secondary school. There needs to be a clearer description of these meanings and techniques appropriate to Year 6 and a better build up. (Consultation no. 92) • Ratio is potentially confusing (for example the potential confusion arising from translating an external ratio of 2:3 as a 'fractional part' 2/3 instead of 2/5 is only one of the potential problem). The primary curriculum should focus on laying the multiplicative foundations for ratio to be learned in KS3. (Workshop)
Year 6 – Numbers – Ratio and proportion	**276**	• Recognise equivalent ratios and reduce a given ratio to its lowest terms [276]	• This does not provide an essential basis for further mathematical study. This objective requires a deep understanding of algebraic and proportional reasoning. (Anonymous)

Year 6 – Numbers – Ratio and proportion	**279**	Ensure pupils practise solving a wide variety of problems so that pupils are taught to apply ratio and proportion flexibly.	• Note that proportion here is only mentioned in the Notes and Guidance and not the Programme of Study. (Consultation no. 16)
Year 6 – Numbers - Algebra	**280**	• Solve linear missing number problems, including those involving decimals and fractions, and find pairs of numbers that satisfy number sentences involving two unkno.wns	• This is a low expectation: Currently many children in Y1 'solve linear missing number problems'. (Consultation no. 49 and 68) • Much more guidance is needed as to why and how letters can be used to increase the efficient solving of some type of problems. (Consultation no. 85)
Year 6 – Numbers - Algebra	**280 and 284**	• Solve linear missing number problems, including those involving decimals and fractions, and find pairs of numbers that satisfy number sentences involving two unkno.wns [280] • Pupils should also write missing number problems algebraically; for example, 2x – 4 = 8 therefore 2x = 12 therefore x = 6 or finding missing lengths in perimeters and missing angles at a point. Pupils should also find possible solutions for equations with two unknown variables, for example x + y = 5 includes solutions x = 1 and y = 4, x = 2 and y = 3.	• This is overly ambitious for many in Y6. Formal algebra is best left to KS3. Using algebra to express generality and number relationships will be appropriate for some but needs to be related to context. (Consultation no. 54) • The mathematics in the curriculum should be written correctly i.e. variables should be italicised. Using x as a variable should be avoided because of possible confusion with ×. (Consultation no. 54) • The foundations for algebra need to be developed in the primary school by working on noticing and explaining patterns and structure. (Consultation no. 54) • This is a potentially large body of techniques which most secondary students take several

			years to learn. These are not likely to be well-taught at primary level and a better preliminary to algebra are the ideas of pattern and structure, some of which, such as inverses are already included. There is a danger of including too many missing number problems as pupils believe that letters always stand for specific unknown numbers rather than variables. (Consultation no. 92)
Year 6 – Numbers - Algebra	**282**	Generate and describe linear number sequences, including those involving negative and decimal numbers, and proper fractions e.g. 1.4, 1.1, 0.8.	• This is a low expectation: Currently many children in Y1 'describe linear number sequences'. (Consultation no. 49 and 68)
Year 6- Number- Algebra	**284**	Pupils should also write missing number problems algebraically; for example, 2x – 4 = 8 therefore 2x = 12 therefore x = 6 or finding missing lengths in perimeters and missing angles at a point. Pupils should also find possible solutions for equations with two unknown variables, for example x + y = 5 includes solutions x = 1 and y = 4, x = 2 and y = 3.	• Implies that only integer solutions are possible. Either it should say 'find integer solutions' or be left for secondary specialist teachers. (Anonymous) • There is substantial research to show that making the jump from finding missing numbers or guessing number pairs using number facts is not as simple as this statement suggests and can create long-lasting misconceptions about the nature and use of algebra. Either this statement needs splitting into two or three more informative statements, or the shift to expressing equivalent expressions algebraically to find values for variables needs to be left for specialist teachers in KS3. (ACME)

Year 6 – Geometry and Measures -	**287**	Recognise, describe and build simple 3-D shapes, including making nets	• This statement has a great deal of prior knowledge and skills implied. Clearer guidance relating to progression of ideas and concepts would be beneficial. (Consultation no. 60)
Year 6 – Geometry and Measures – Properties of shapes	**288**	Describe properties of cuboids and other common 3-D shapes including prisms and identify parallel planes and symmetries	• This is inappropriate and should be left to KS3/4. This is an inconsistent development from line symmetry in Y4 **[170].** (Workshop + Consultation no. 54) • This is well beyond the essential core knowledge. (Consultation no. 2)
Year 6 – Geometry and Measures – Properties of shapes	**290**	Find unknown angles involving angles at a point, on a straight line, in a triangle (180^{o})**,** in a quadrilateral (360°) and vertically opposite angles.	• This is inconsistent with **[228]** which correctly states 'angles at a point on a straight line'. (Workshop + Consultation no. 54) • Should say 'angles at a point, at a point on a straight line, in a triangle, etc.... (Workshop)
Year 6 – Geometry and Measures – Properties of shapes	**292**	Ensure pupils also describe properties of shapes and explain how they derive unknown angles and lengths from known measurements.	• Reference could be made to use of scale drawings here and the statement should distinguish the use of scale drawings from the use of logical reasoning. (Workshop + ACME+ Consultation no. 54)
Year 6 – Geometry and Measures – Position, direction, motion	**293, 295**	• Describe positions on the full coordinate grid (all four quadrants) • Ensure pupils practise drawing and labelling a pair of axes in all four quadrants and drawing pairs of axes. This extends pupils knowledge of one quadrant to all four quadrants, including the use of negative numbers.	• Recommend working on axes in just the first quadrant at this stage and leave all four quadrants for KS3. This requires fluency with numberline as the model of real numbers. (Workshop + ACME + Consultation no. 54)

Year 6 – Geometry and Measures – Position, direction, motion	**294**	Construct, translate and reflect simple shapes on the coordinate plane.	• Transformations (i.e. operations on shapes) are best left to the KS3 PoS given their conceptual challenge. Children should create designs and patterns that involve repeating shapes. (Workshop) • Bryant (2009: 34) reports on the difficulties children, including many in secondary schools, experience with the transformation of shapes. (Workshop)
Year 6 – Geometry and Measures- Measures	**297**	Use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, including between miles and kilometres	• Suggest leaving imperial measures to KS3 other than everyday equivalences e.g. knowing that distances can be measured in km or miles, in metres or yards, capacity in litres or pints, etc. (Workshop + Consultation no. 54)
Year 6- Geometry and Measures- Year 6 – Geometry and Measures- Measures	**299, 304**	• alculate the area of parallelograms and triangles [299] • Ensure pupils use the formula to calculate area of a triangle and a parallelogram. Include identifying the base and its corresponding height. Exclude finding the base or height of a triangle given its area. [304]	• Recommend leaving parallelograms to KS3 to avoid confusion between area as 'counting squares' and area as 'multiplying sides', neither of which is appropriate here. In KS3 teachers can use these topics to develop a more robust knowledge of area which will lead on to more complicated mensuration, trigonometry and calculus. If it is to be kept here is should read "Identify a suitable base and height for triangles and parallelograms in any orientation; understand how to use the formulae for areas of triangles and parallelograms in any orientation.'. (Workshop + ACME + Consultation no. 54)

Year 6 – Geometry and Measures- Measures	**300**	Recognise when it is necessary to use the formulae for area and volume of shapes	• This probably means to solve problems involving shapes which require the use of formulae to find area or volume but it would gain a rewording. (Workshop + Consultation no. 54)
Year 6- Geometry and Measures- Measures	**301**	Calculate, estimate and compare volume of cubes and cuboids using standard units, including centimetre cubed (cm^3) and cubic metres (m^3) and extending to other units, such as mm^3 and km^3	• It is not clear whether kilometre-cubes are really standard units that can be estimated. This should be reconsidered. (Workshop) • Centimetre cubed (cm3)' should be read as 'cubic centimetres'? (Workshop)
Year 6 – Geometry and Measures- Measures	**305**	Pupils can be introduced to other compound units for speed such as miles per hour and apply their knowledge in science as appropriate.	• Compound measures such as speed are conceptually very demanding and are best left to KS3/4. This statement is presumably a follow on from **[132].** (Workshop + Consultation no. 54)
Year 6- Geometry and Measures	**306- 307- 308**	• Draw, read and interpret line graphs and use these to solve problems [306] • Use and interpret averages including mean, median and mode and solve simple problems using different kinds of averages. [307] • Ensure pupils understand and use a range of graphs such as: temperature time, distance-time and currency conversion. They should apply their knowledge in science and other subjects as appropriate. [308]	• Mean, median and mode: there should be information in the notes and guidance about why you need to teach them. (Workshop) • There is no mention here about when line graphs might be used. It would be appropriate to introduce scattergraphs here. (Workshop) • The introduction of measures of average separately from understanding data collection makes no sense. (Workshop) • There are links with algebra in **[308]** which should be highlighted here. (Workshop)

			• There is a typo in **[308]**: temperature-time. (Workshop)
Year 6- Geometry and Measures - Data	**307**	• Use and interpret averages including mean, median and mode and solve simple problems using different kinds of averages.	• Children should understand the appropriateness of different averages, not just be able to calculate them. They should be explicitly encouraged to work with both discrete and continuous data; they should appreciate lots of different approaches to calculation, etc. (Consultation no. 16)
Year – Geometry and Measures - Probability	**309-310**	• Use the language associated with probability such as certain, equally likely, unlikely, impossible and use this to describe the likelihood of particular events. • Ensure pupils understand and use the notation of probability and fractions; for example, the probability of rolling a 3 on a six-sided die is P(3) = 1/6, with the numerator showing the number of given outcomes and the denominator the number of possible outcomes.	• The N&G here do not match the PoS. The ideas and language in the PoS could be introduced at a much earlier point, even in KS1 with appropriate contexts. (Workshop + consultation no. 16) • The calculation of probability, and particularly the expression of this as a fraction, is a much more advanced idea. (Workshop) • The use of the probability scale from 0 to 1 is best left to KS3. Probability is a very challenging concept. Primary children are much better to work with practical contexts and develop the vocabulary of chance. (Consultation no. 54) • There is the implicit assumption that the outcomes are equally likely. In the Notes, this should be pointed out. It requires either: talking about a ‘fair’ die (in the example) or (in general) saying the outcomes must be ‘as

			likely' to occur. Equally likely outcome situations represent a very narrow range of events. (Consultation no. 24). • Welcome the fact that probability and averages are not taught before Year 6. (Consultation no. 63)