Summary of ACME questionnaire responses received on the draft National Curriculum for primary school mathematics

Section 1: Major themes emerging from the responses to the consultation

1. There was much consensus amongst respondents from different sectors.
2. The proposed aims were welcomed but the proposed programme of study was not seen to reflect the aims.
3. Respondents said that more attention should be given in the programme of study to understanding, problem solving, reasoning and application, rather than so much to formal written methods of calculation.
4. There was demand for more connections between different areas of mathematics and between mathematics and other subjects to be made.
5. Respondents reported that the implied pedagogy (transmission of knowledge, passive learning, recall and practising) was not appropriate.
6. Many respondents predicted that trying to do “too much too soon”, particularly in formal written calculations, would be counter-productive.
7. Some respondents commented that the programme should be brought up to date, to recognise the opportunities and demands of the 21st century.
8. The lack of attention to recent and relevant research about how children learn mathematics was regretted, as was the conflict between this draft and recent Ofsted reports on teaching and learning mathematics.
9. The dangers, as well as the benefits, of referring to the curricula of other jurisdictions were recognised.
10. Nearly all respondents reported that the proposed curriculum would have a negative effect on pupils’ mathematical competence and understanding.

Section 2: Detail of the responses to the ACME questionnaire

ACME published a detailed questionnaire about the draft programme of study for mathematics on Monday 18 June 2012. The committee received 98 written submissions (see appendix). The submissions came from a spectrum of individuals and organizations, including teachers, schools, Local Authority advisors, education researchers. Several responses were received from mathematics organizations including:

- the Association of Teachers of Mathematics (ATM) and the Mathematical Association (MA),
- the London Mathematical Society (LMS),
- Mathematics in Education and Industry (MEI), and
- the Royal Statistical Society (RSS).

This report has been prepared independently of ACME and goes through the ACME response questionnaire question by question, highlighting the major themes and quoting representative comments from respondents.

1. Aims

Question 1.1

1.1 In your view, do these aims set out the right teaching and learning priorities for mathematics? If not, how could they be changed?

In general, questionnaire respondents and those participating in the web seminars and the 3 July London workshop were happy with the stated aims and the introductory paragraphs within which the aims appear. [1]62%[2] said that these aims as they stood set out the right teaching and learning priorities for mathematics, reflecting current good practice and setting high expectations for primary school pupils' mathematical understanding and competence.

16%[3] of respondents said they were not happy with the aims; some of these were criticising the way that the aims were manifested in the programme of study rather than the aims themselves.

The following were among suggestions to improve the aims:

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[1] Throughout the report, the numbers of respondents giving a response that could be identified as 'yes', 'no' or 'yes/no' were noted and percentages of the total number of such responses were calculated. The total is indicated in a footnote in each case.

[2] Out of a total of 74

[3] Out of a total of 74

- A primary school teacher said that the aims *"could be further enhanced by including reference to the need to present the mathematics curriculum in an enjoyable, interesting way that is relevant to real life"*; other respondents echoed this view.
- Others suggested that the relative importance of understanding needed to be raised; a large minority mentioned this specifically in their responses. One respondent (an initial teacher trainer) suggested that the first aim would be better expressed as: "*pupils become fluent in mathematical techniques, methods, recall of number facts and algorithms, and have a good understanding of the underlying concepts*". Another respondent working with a group of primary schools suggested that the order of the aims should be changed.
- A mathematics education research group criticised the sentence 'Pupils should therefore be taught to practise and then apply their mathematics to a range of problems' in one of the introductory paragraphs. *"There is no justification for this separation and ordering of activity, nor for the exclusion of conceptual activity; there is some evidence that pure practice of algorithms before use in related word problems results in pupils not being able to remember or apply what they have been taught. Teachers who build conceptual development by connecting ideas have been shown to be more effective in producing learning."*
- The same group suggested replacing *'appropriate written algorithms and mental methods'* by *'appropriate methods'* in the first aim.

Question 1.2

1.2 In your view, should the aims – either those set out above or any alternative aims you propose – be made more explicit in the main content of the Programme of Study? If so, how?

In response to the question about whether the aims should be made more explicit in the programme of study, 96%[4] said that they should. Respondents stated that there was a contrast between the introduction to the proposed curriculum and the detail of the programme of study. There were concerns that teachers would not be encouraged to teach the curriculum as specified by the aims unless the programme of study was specified to match those aims and integrate the intended curriculum throughout.

Respondents' disappointment with the lack of reflection of the aims in the programme of study included the following.

- *"The PoS does not reflect these aims in any way!"* (Primary consultant)
- *"I wholeheartedly agree with these aims. Unfortunately, I am not convinced that the Programmes of Study will help ensure these*

[4] Out of 56 respondents

aims are fulfilled." (Primary teacher)

- *"After reading the entire document I felt quite disappointed from such a positive and enthusiastic start, that the programme of study did not support or reflect the aims."* (Primary teacher)
- *"There is little evidence of these aims … in the notes and guidance column giving examples of what is to be taught. If they were interwoven with the skills teaching, it would be clearer to teachers that these aims need to be addressed in every maths lesson."* (Primary teacher)
- *" … the programme of study does not encourage or support teachers to implement [the aims]"* (Primary teacher)
- *"The use of words such as conjecture, explore, investigation, understand, generalise, connections need to be appearing throughout the document not just at the start!"* (Primary teacher)
- *"[We are] therefore disappointed to find that the detailed draft Primary Curriculum does not in any way reflect these aims. The new draft especially lacks a coherent vision regarding problem solving and the implementation of this curriculum as it stands would lead to a serious lack of developing elementary problem solving skills in primary school children, which would be difficult to rectify in their subsequent education."* (Learned society)
- *"The problem is that the Programme of Study does not relate in any balanced way to the Aims. While it is not easy to relate the two explicitly, an analysis of the statements would reveal that there is too much focus on algorithmic fluency and not enough on concepts, problem-solving and reasoning."* (Mathematics education research group)

As indicated by the above quote, a number of respondents indicated concern that the programme of study focused almost entirely on the first aim (fluency) rather than on the second and third. The frequent occurrence in the programme of study of words such as 'practise', 'write', 'recognise' and 'recall' rather than 'explanation', 'reasoning' and 'understanding' and 'problem-solving' was indicated as one reason for this. (A group of primary teachers pointed out that the word 'understand' appears only nine times in the entire document and that these all occur in the notes for guidance not in the actual programme of study.)

Respondents were concerned that in the programme of study 'fluency' seemed to refer to calculations only and sought a wider definition.

Many respondents said that there should be more emphasis on using mathematics and applying it to real problems; the mention of *'word problems'*, suggesting standard textbook exercises, was seen to be not sufficient to encourage good practice in application and problem solving. One contributor to a web seminar said that using and applying mathematics should be '*a golden thread running through everything'*.

There was similar regret about the omission of reference to investigative mathematics; respondents indicated that that both their experience as

teachers and research by academics demonstrated the positive effects on the development of mathematical understanding and confidence of pupils exploring mathematics for themselves.

One primary teacher said
"… I think teachers will feel that the government do not actually value investigations or problem solving, which automatically feed into mathematical reasoning and conceptual understanding."

In this respect, as in others, respondents reported that the draft programme of study conflicted with the advice given in the recent Ofsted report *Mathematics: Made to Measure.*

A primary teacher was one of many respondents who referred, in this part of the questionnaire and others, to this report*:*

"*I quote from the most recent Ofsted report (2012):* '*The responsibility of mathematics education is to enable all pupils to develop conceptual understanding of the mathematics they learn, its structures and relationships, and fluent recall of mathematical knowledge and skills to equip them to solve familiar problems as well as tackling creatively the more complex and unfamiliar ones that lie ahead.' The writers of these proposals would have done well to read this report, which paints a far from rosy picture of our mathematics education, but repeatedly refers to understanding as being of critical importance, of the vital need to improve students' problem solving and investigative skills and makes statements such as:* '*too much teaching concentrated on the acquisition of disparate skills that enabled pupils to pass tests and examinations.' "*

A secondary mathematics teacher referred to Sir Michael Wilshaw's introduction to the report:

> Then, we will raise ambition for the mathematics education of all pupils by placing greater emphasis in school inspection on:
>
> - how effectively schools tackle inconsistency in the quality of mathematics teaching
> - how well teaching fosters understanding
> - pupils' skills in solving problems
> - challenging extensive use of early and repeated entry to GCSE examinations.

The teacher comments: *"In this context the draft new primary curriculum seems to be at odds with developing and preparing pupils for secondary mathematics, with little evidence or emphasis on understanding and applying problem solving skills in any context other than 'practice'.*
For many primary teachers (many of whom will not be maths specialists) the language and tone of the draft mathematics curriculum will support a narrow interpretation of maths as prescriptive rote learning of limited algorithms at great speed with little emphasis on understanding and developing problem solving skills. This could lead to narrower teaching and learning styles geared

to ‘exam cramming’. Not only would this be counter to a great deal about what we know regarding effective learning through research but also some of the best practice in the world e.g. Singapore -(where I believe understanding and problem solving skills are actually key). So many mixed messages. Please don't destroy the good things that have been done in our attempts to continue to improve our children’s learning of mathematics.”

Several respondents expressed the need for mathematical talk on the part of pupils to be an important part of the programme of study, concerned that there was little, if any mention of this. Some respondents agreed that oracy was very important but were not sure that it should be specified in the National Curriculum.

A researcher into primary mathematics teaching was one of those concerned about mathematical relationships: *“No mention is made of mathematical relationships in the introductory paragraph to the Key Stage 1 Programme of Study. This risks leaving the important concept of symbolic equivalence merely implicit throughout the curriculum, and it might subsequently be overlooked by teachers and resource designers. I suggest amending the second sentence to read: This should involve working with numerals, words, the four operations and the relational symbols (=, < and >), including with practical resources (concrete objects, measuring tools, etc.)."* S/he provides similar suggestions for the Key Stage 2 programme of study. Others said that, although working with mathematical relationships was very important, the use of < and > symbols was not appropriate in Key Stage 1.

2. Structure, expectations and challenge

Question 2.1

2.1 Is the proposed National Curriculum for primary mathematics set out in such a way that you can **understand** what expectations are intended for each key stage (for example, in its balance between over-specificity and insufficient detail)? Please be specific about which parts of the presentation are supportive of teachers and other users, and suggest alternatives if you find aspects of it less helpful.

- Are the key mathematical ideas clear and central to the PoS?
- Is there sufficient detail?
- Is the level of specificity right?
- Is the wording clear and helpful?
- Does it give sufficient information to support the construction of an effective scheme of work?

The responses to this question did not reveal key trends; respondents wrote about issues of concern with the programme of study but not necessarily as direct answers to the questions asked. Of those whose answers revealed preferences, approximately half reported that they understood the

expectations, 30%[5] that the key ideas were clear, 25%[6] that there was sufficient detail, 15%[7] that the level of specificity was right, and 30%[8] that the wording was clear.

A mathematics education research group highlighted some of the difficulties: *"The draft reads like a list of fragmented Attainment Targets rather than a coherent Programme of Study, but even then the Attainment Targets referred to are mainly of a narrow type involving specific disconnected knowledge of symbols or algorithms. Attainment Targets should properly be concerned with specifying what pupils should be able to do and not how they should do it. (For example, pupils should be able to both estimate the numbers of children in a school given the number of classes and calculate the exact number given the class sizes - the methods chosen should properly depend on the numbers and should not be specified nationally as long as they are effective and efficient.)"*

Question 2.2

2.2 Does the content outlined in the draft mathematics curriculum set the right expectations for 5 to 11 year-olds, taking account in particular of the expectations set in high-performing jurisdictions? If not, what expectations do you think need to change and why?

The response to this question was clearer; 85%[9] wrote that the draft did *not* set out the right expectations for 5 to 11 year olds, often because the expectations were thought to be too challenging and/or outdated and not useful.

For example, one primary teacher expressed the views of many that *"It seems unlikely a Year 1 child could be expected to move from early learning goals to being able to find ¾ of an amount."* In general, the requirements on calculating with fractions were reported to be *"aspirational"* and it was pointed out that even some able pupils about to take GCSE are unable to do such work successfully and confidently.

A key theme, mentioned by many respondents, was the potentially negative effect of trying to do too much too soon. A primary consultant said *"The programme of study is unnecessarily 'front-loaded' with relatively difficult abstract ideas, apparently in order to leave large amounts of later time available for verbal/symbolic 'practice'. When abstract ideas are presented prematurely to children, however – as in the case of 'fractions as objects' in KS1 (statement [24]) – this approach is self-defeating; the later 'practice' effectively reinforces poor understanding and encourages empty ritual."*

[5] Out of 23 respondents
[6] Out of 8 respondents
[7] Out of 13 respondents
[8] Out of 7 respondents
[9] Out of 60 respondents

A mathematics education research group suggested a way forward: *"There are a number of places where the expectations listed are significantly higher than those we have at present. Yet many pupils still fail to master those we have now. It is not clear how piling yet more on will help those pupils. It may well be that there are places where expectations can be raised and that this will lead to better achievement but we feel that some random controlled trials are needed to test this assumption before testing it out on the whole nation. Otherwise there is the risk that if there is more work to cover pupils will end up with a less sure grasp of it - on balance we believe that more consolidation is needed of what is on the current programme of study in primary key stages."*

A primary consultant pointed to the imbalance in the expectations: *"The programme of study seems to set some raised expectations but these are very skills/content based and without the balancing expectation for problem solving/reasoning/conceptual understanding may not produce effective mathematicians. There seems to be a drive to accelerate through skills such as formal written methods, manipulation of fractions at the expense of understanding. Many children will achieve the mechanics but may not have a secure understanding of the concepts – others will fall further behind."*

These were concerns expressed by a majority of respondents, one of the over-riding themes of this consultation. All categories of respondent were adamant that pupils had to be given the opportunity to apply what they knew to a variety of situations in a flexible and informal way to develop understanding and fluency without being expected to use formal written methods. This was one of the many areas where respondents referred to relevant research about how children learn as well as recent reports such as those produced by Ofsted and the Williams review. As already mentioned, the mismatch between this draft curriculum and Ofsted's *Mathematics: Made to Measure* was frequently highlighted.

Some respondents expressed positive views and experiences: one primary teacher wrote, *"I feel there should be more emphasis on children exploring mathematical conceptions to deepen their understanding. In my school in key stage 1 we place a great deal of emphasis on exploration to make connections and the results and abilities are outstanding; the children LOVE maths and work with great effort and enjoyment."* A consultant wrote, *"Children's ability to reason and make connections is underestimated by the draft curriculum, so the expectations in those areas should be increased."*

A joint submission by two professional mathematics teachers' associations highlighted particular areas of mathematics thought to be incorrectly placed in the primary curriculum: *"The expectations set out in the draft programme of study are too high. There is a rush to introduce formal representations, which contradicts research evidence that emphasises the importance of developing secure conceptual understanding* (research references were included here)*. Roughly speaking the expectations for the end of Y4 are similar to the expectations for Y6 currently, and many of the expectations for upper Key Stage 2 include things which are best left to secondary, e.g. use of the*

probability scale, coordinates in four quadrants, the compound measure: speed, scaling of area (without any previous reference to scaling of lengths), formal algebra, volume and transformations. These topics are left until lower secondary school in the high performing jurisdictions without exception as far as we can ascertain. Care also needs to be taken to ensure coherence with the programme of study for primary science. It is unclear why major changes have been made to the science curriculum given the consistently high performance of English students in international comparisons."

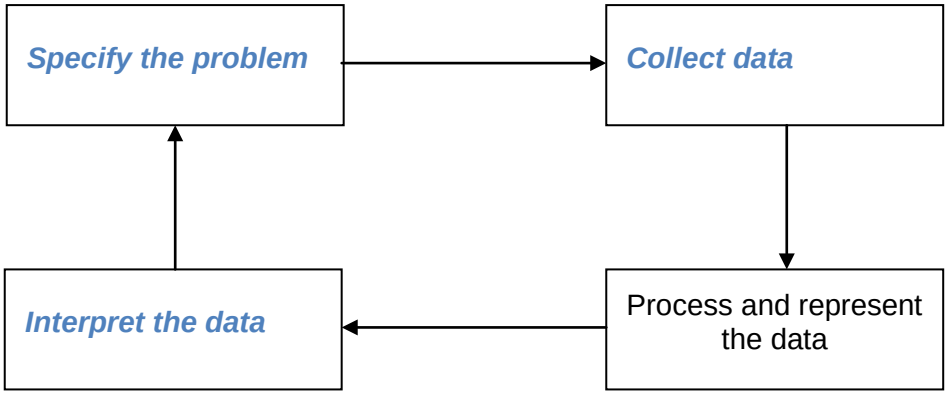
This submission also included a list of welcome inclusions in the draft programme of study: *"There are some things which are welcomed: a strong focus on counting; the importance of reading and spelling mathematical vocabulary; the emphasis on the link between division and fractions; using the correct language for features of 3D shapes – faces, edges and vertices; and separating perimeter from area."*

The reference to the science proposals was not isolated; several other respondents wrote of the need to align the science and mathematics programmes of study so that pupils could consolidate their understanding and not be confused by conflicting expectations.

Many respondents regretted the lack of reference to models and images, such as the number line, to support children's learning of mathematics. One respondent pointed to the use of Cuisenaire rods as a powerful means of developing understanding of fundamental algebraic ideas from Year 1 onwards. A consultant said *"In learning from 'high-performing jurisdictions' it is important to study everything about their mathematics teaching, not simply when they introduce which abstract ideas. The high-performing jurisdiction of Singapore, for instance, is noted for widespread use of a 'bar model' in its mathematics teaching – an attention to the visual nature of mathematical communicating and thinking entirely and critically unrecognised in this current draft curriculum for England."*
Instead, the suggested pedagogy in this draft appears to consist of practice of formal methods of calculation and memorisation of vocabulary and properties of shapes.

Some people pointed out that expectations were too low in some areas such as mental calculation, problem solving, algebraic thinking, ratio and proportion, and data handling.

A curriculum development group said that the data handling cycle shown below should be included in the Key Stage 2 curriculum; they also recommended the use of a problem-solving cycle.



The data handling cycle

Most respondents who commented on the inclusion of Roman numerals and numbers in binary form said that these could be very interesting to investigate, perhaps as part of some cross-curricular work, but that they did not have a place as something that all pupils should learn and on which they may be assessed.

Item 49, column methods of addition and subtraction without the need for exchange between columns, was thought to be likely to lead to the subtraction misconception of just find the difference between the digits in the column.

Respondents did not see the need for pupils to learn tables up to 12 x 12.

Year 3 was said to be too young for pupils to be drawing circles and arcs with a pair of compasses and, in any case, the use of being able to do this questioned.

Many respondents wrote about year on year specification of the programme of study and few were happy with this, preferring the greater flexibility of specification by key stage. There was concern about pupils who were not able to complete the work at the appointed stage or who were ahead of others of the same age.

It was suggested by a number of respondents that in Year 1 children could learn, rather than numbers from 11 – 20, larger numbers with one significant figure, because of the easier vocabulary of such numbers compared with the teens.

The use of standard measures in the early primary school years was stated to be inappropriate; respondents reported that children need to explore the dimensions of objects by comparison and by the use of non-standard units before moving on to formal measurement.

Question 2.3

2.3 On balance, is the level of challenge envisaged appropriate for the range of young people in all the key stages covered? If not, please be specific about what you think should be changed and why.

The responses to this question covered the same ground as those to the previous one; respondents did not appear to distinguish between expectations and challenges. 87%[10] said that the level of challenge was not appropriate.

Question 2.4

2.4 In light of the new duty on schools to develop and publish their school curriculum for mathematics by academic year, is the key stage and academic year layout of the proposed national curriculum helpful? If not, what would be more useful for teachers?

One third of the 27 respondents who expressed a view one way or the other indicated that the layout would be helpful in publishing their curriculum. Many responses focused not so much on publishing the curriculum, but about aspects of the curriculum itself. As mentioned earlier, there was some considerable feeling against the strict year by year format, one teacher saying that may encourage some teachers to think more about what to teach than why to teach it, and others commenting on the difficulties of managing both ends of the attainment spectrum if pupils were expected to learn at the material by the end of the year. Some highlighted potential difficulties in schools with mixed-age classes.

Several teachers expressed the wish for schools to publish information about how the different areas of mathematics progressed through the years and how they linked together. Some said that schools should be provided with this information; others that schools could expect to develop their own schemes of work, though some may find this hard if they have little specialist mathematics expertise.

The professional association representatives suggested the following, which may satisfy the needs identified above: *"In order to support schools to plan from the Programmes of Study, we would recommend that the guidance part of the pages is used to provide pedagogical support and subject knowledge for teachers. This could include how manipulatives can be used to support conceptual understanding. This guidance could also suggest opportunities to: reason mathematically, follow lines of enquiry, solve problems (including real life problems), persevere and be curious (as set out in our aims); recommend useful contexts for exploring the mathematics; identify links between this and other areas of mathematics / other subjects; and also suggest some high quality, freely available / common research-backed resources to support learning (for example, the content of the NRICH website). This is in line with current practice in Flemish Belgium and Hungary."*

[10] Out of 44 respondents

Some respondents expressed a preference for information by level rather than by year.

Question 2.5

2.5 What other aspects of the proposed curriculum either support or hinder the development of a school's mathematics curriculum? In particular, what aspects of the draft could be left more to local professional judgement?

In answer to this question, a few respondents came up with what they considered to be **supportive aspects** of the proposed curriculum.

- One group welcomed the statement on inclusion in the introduction (though they were not sure how it would be implemented); one teacher mentioned the helpful emphasis on knowing tables and another said, *"I think that the PoS is a very helpful document – it needs to be explicit."*
- A primary school was pleased to see the division of Key Stage 2 into two parts and said that the Year 5 percentages and equivalent fractions sections were clear.
- A local authority representative said, *"The removal of levels is welcome but there needs to be clarity about what will replace them and how children's attainment is to be reported."* Other respondents, however, regretted the removal of levels.
- A primary adviser anticipated that the precision of the draft programme of study would *"…help weaker teachers to identify quite clearly where they are unsure and need advice/CPD."*

Suggested **hindrances**, in addition to those mentioned elsewhere, were:

- over-prescription, particularly in calculation methods
- the requirement to record calculations even in Year 1
- lack of emphasis on mental methods *("If you know how to calculate mentally, you can derive new number facts yourself!"*)
- lack of a year-by-year progression in problem solving and reasoning *("The over emphasis on repetition, drill and practice hinders the achievement of the aim of learning to reason mathematically"*)
- lack of reference to the need for children to explain mathematics
- *"the lack of a unifying using and applying strand and any reference to assessment for learning practices"*
- lack of emphasis on thinking skills and understanding
- no mention of cross-curricular mathematics.
- *"Lack of guidance about appropriate pedagogies based on educational research will create extra work for schools."*

These were common themes, mentioned by many respondents.

The most common suggestion for what could be **left to local judgement** (possibly within partnerships) was calculation strategies, perhaps because

this was the aspect of the proposals that most respondents were unhappy with.

One respondent, who did not reveal his/her role, said, *"Specific methods for calculations should be left to professional judgement. Children need to be taught to look at the numbers and choose the most appropriate method for the context."* Another asked hopefully: *"Are we to assume that 'formal methods' means traditional British algorithms only or can we use the formal methods used by other countries, as demonstrated by Ian Thompson (1998)?"*

A consultant was clear that *"The specification of column arithmetic is not helpful. It may appeal to the electorate but this is to suggest that there is no expertise in education and that a public view can hence prevail. Applied to medical practice this type of decision making process would result in a large number of deaths."*

3. Implementation

Question 3.1

3.1 Would you anticipate that the teachers you are familiar with will be able to implement this curriculum in a coherent and engaging way, so as to result in the vast majority of young people achieving a robust and confident mastery of primary mathematics? If that is true only in part, or for some year groups, please indicate which, and if possible, why.

70%[11] of respondents did not think that the proposed curriculum could be implemented in a coherent and engaging way. 11%[12] thought that it could.

Some of the implementation concerns focused on the lack of necessary subject knowledge in primary teachers, particularly for formal 'long division' algorithms, fractions, Roman numbers and the binary system. Some respondents reported that some teachers, such as those new to the profession who did not have confident mastery of the curriculum, would have difficulty in working flexibly with pupils to help them develop understanding and would tend instead to teach in a fragmented way. A teacher training partnership respondent's view was, *"Teachers do not often understand the links between concepts (e.g. division links to fractions, fractions link to percentages, weight links to mass). Either these links need to be made far more explicit in the guidance OR there needs to be substantive training to get these ideas correct at last."*

Others considered that the demands of the curriculum were too great, as indicated by this primary teacher: *"If the curriculum is too challenging, particularly in Years 1- 2 when children are still developing the key concepts and mental images that underpin all future learning, even the best teachers in*

[11] Out of 35 respondents
[12] Out of 35 respondents

the world may struggle with getting children to understand. You can set a 10 month old the target of walking because a few children can walk at 10 months, but this does not mean that the average 10 month old is developmentally ready to do so. The same applies in all areas of learning."

A response from a mathematics education research group highlighted other perceived difficulties in implementation: *"There are some parts of this proposed PoS that we would find it difficult ourselves to present in a coherent and engaging way e.g. learning the symbols for multiplication and division before you can understand the operations well enough to be able to carry them independently with real objects. Much of this programme of study is expressed as fragmented knowledge and procedures - it will require teachers with a very clear overview of mathematical concepts and connections and a lot of time to implement it in a coherent and engaging way. Yet many primary teachers have weak knowledge of mathematical content and of mathematical didactics. Thus we are very concerned that children will experience the curriculum as disconnected bits which make little sense, which does not make it any easier to remember them."*

A primary teacher wrote, *"I feel that the implementation of this curriculum will struggle to achieve 'robust and confident mastery of primary mathematics' based on the fact that there is no mention of open-ended tasks or investigations."*

Question 3.2

3.2 What would be the practical implications for schools of teaching this mathematics curriculum, including:

- Training requirements in the short term?
- Ongoing CPD in the medium term?
- Resources?
- Teaching time?
- Other support?

Respondents were very clear that training was the **major implication of implementation of the curriculum**; 98%[13] indicated the need for this. Much smaller numbers highlighted the need for extra teaching time, resources and other support. Several respondents mentioned the need for time and training to construct school schemes of work from the proposed curriculum.

An infant teacher wrote about training needs: *"I am sure that all teachers I know will do their best for their classes. However, I feel strongly that a new curriculum cannot be taught effectively without training. Many teachers have only been teaching since the implementation of the NNS and are used to having a prescribed and planned curriculum and therefore will need some*

[13] Out of 44 respondents

guidance to create a full curriculum from a list of objectives. This is going to present problems as in many authorities there are no longer staff to carry out this training due to government cuts."

Many respondents identified specific needs, for example: *"In the short term, schools will need support on interpreting and implementing the content of the National Curriculum document and on making the curriculum their own. This will include: understanding how the needs of low attainers are to be met; what is to replace levels; what planning might look like to ensure that whole-class mastery/readiness to progress happens; what mastery looks like. In the medium term, teachers will need quality support with their mathematics subject knowledge, their understanding of progression through the objectives and suitable teaching approaches to ensure that they are covering the key knowledge and skills that underpin learning. In addition, recent reports by the Cambridge Primary Review and Sir Peter Williams have outlined what a good curriculum should look like and their recommendations are at odds with the content of this proposed curriculum. Teachers, particularly those who have been recently trained, are familiar with these reviews and have been trained to implement the recommendations."* (Professional association group)

Others, from schools, local authorities and higher education, said that more work was needed before moving ahead:

- *"We have to ensure that the curriculum itself is appropriate before we could make any sensible suggestions about training needs."*

- *"There is a need to pull together research on effective teaching strategies for the items in the curriculum. The last time such a review was done was in the 1990s in preparation for the introduction of the National Numeracy Strategy. What we need now is a thorough review of the available research evidence on what works well. That should begin as soon as possible and be funded by government. The outcome of that review of available research evidence should then steer the content of the training that is needed. Train ITE primary maths tutors. Train head teachers as well as mathematics subject leaders. Provide paid time off for teachers to understand the teaching implications of the research review."*

- *"It is worth considering delaying implementation by a year to give time for this necessary quality CPD to be rolled out to all teachers. Primary teachers need to feel confident that they have the necessary tools and that they are enabled to make this curriculum work. These demands will be less if the expectations are made more realistic and the guidance element of the document is more comprehensive."*

A respondent writing on behalf of a learned society also wanted delay: *"…if the final version is to be 'fit for purpose', the publicly declared timetable must be revised (with implementation delayed by a whole year)".*

The suggested **resource needs** included:

- Case studies exemplifying teaching approaches
- Online forums for discussion and sharing ideas
- List of useful schemes and materials
- *"Manipulatives and teaching resources - they should already have [these] in classrooms"*

When commenting on the **teaching time required**, the general view was that at least as much time as at present would be needed:

- *"An hour a day would not be sufficient to deliver the curriculum, especially at year 6."*
- *"Teaching time – will more time be needed to be dedicated to teaching maths? Will this mean less time for other subjects or can it be taught more cross curricular?"*

The needs for **other support** included CPD for teaching assistants, information meetings for governing bodies and for parents and time for network cluster meetings.

4. Attitudes

Question 4.1

4.1 Will the proposed mathematics curriculum have a positive effect on confidence, understanding and interest? If not, why not, and what changes would improve this?

Many of the responses to this question highlighted concerns that have already been discussed. The overall opinion was very negative: 86%[14] of respondents stated that the proposed curriculum would *not* have a positive effect on confidence, understanding and interest.

Some typical comments:

- *"I am sorry to say that the new draft curriculum seems to be a 'drill and kill pedagogy' where the enthusiasm for the subject does not filter through. The curriculum in its current draft state is NOT fit for purpose and will end up developing further negative attitudes towards the subject."* (Role not identified)
- *"No – where is understanding and enjoyment emphasised in this document?"* (Local authority representative)
- *"In its present form – no. Too much emphasis on calculation that will not capture interest and too much emphasis on 'regularly practising' – little or no mention of understanding. For example, the KS1*

[14] Out of 56 respondents

programme of study wants children to write x and ÷ statements and calculate the answer [18] but surely the priority is to understand what multiplication is and what division actually is (eg sharing and grouping). This is true of statement [20] as well – the emphasis always seems to be on how to do something rather than understand what they are doing. How does this relate to Skemp's research of instrumental and relational understanding? Also, there appears to be limited reference to models and images to support understanding e.g. arrays for multiplication." (Primary mathematics adviser)

A small minority indicated that it was up to the teachers to make the curriculum work: *"Attitudes will not fostered by a curriculum alone but how it is taught will. I have mentioned that the challenge of the proposed draft could risk turning pupils off mathematics when this is taught in a clinical way. The programme of study as it stands is an austere document but the teaching of it need not be and this is what will help or hinder pupil attitudes. However teachers may find planning and teaching to these new expectations challenging.*" (Independent primary consultant)

5. Content

Question 5.1

5.1 Overall, is the balance of content set out in the draft mathematics curriculum broadly right? If not, what do you think needs to change?

A very large majority, 94%[15], indicated that the balance of topics was *not* broadly right. The concerns expressed focused mainly on the emphasis on formal written calculating rather than on thinking and reasoning.

This primary teacher's views are typical: *"The new curriculum is heavily biased in favour of number and calculating formally. It does not allow enough opportunity for the thinking that children need to develop to become mathematicians and links ability to reproduce a method with ability to do mathematics. It negates the importance of understanding the methods, and ultimately does not support the aim of developing reasoning skills."*

Another said, *"I would be concerned if 'recording' became as or more important than conceptual grasp and understanding."*

Other comments mentioned the need in a balanced curriculum for: investigative work; money/financial capability; earlier work on data handling including probability; earlier use of calculators; applications including cross-curricular links.

One respondent (role not given) mentioned practice elsewhere: *"The content has not got the balance right between mental and written methods in my opinion or with the provision of models and images which are so important to*

[15] Out of 49 respondents

understanding and central to Singapore maths curriculum. The methods of communication seem to be repeating facts and rules rather than communicating understanding."

A consultant working with primary schools was one of those suggesting that algebraic ideas should be introduced earlier than indicated in the programme of study: *"The balance is skewed towards counting, drill and practice which pupils can find difficult and which is often demotivating to teachers and pupils. These risks can be avoided by adding … early algebra.... The classroom time saved in achieving the goal of mastery of number bonds and product tables may be used to bring forward mathematical content from the early years of the secondary curriculum."*

Question 5.2

5.2 Does the proposed curriculum sufficiently promote connections between key mathematical ideas, and between mathematics and other curriculum areas? If not, how could this be achieved?

Again, a very large majority, 98%[16], said that the proposed curriculum did *not* promote connections between key mathematical ideas.

Respondents commented on the almost total lack of reference in the proposals to links within mathematics, between mathematics and other subjects, and between mathematics and life and work. They said that links should be made explicit within the programme of study.

This came from an adviser: *"The proposed curriculum in its current form does not promote connections in any way – this could be something the right hand side column is used for. For example, within measures make explicit the link to place value, within time – make links to fractions and 5x table. It doesn't help by giving topics a specific heading of their own. For example, decimals should be incorporated within 'Number and Place Value', proportion should sit alongside 'Fractions'* " and this from a mathematics education research group: *"… the proposed programme of study is very fragmented, with few connections expressed. We need some means of ensuring that more sustained periods of teaching of specific topics occurs, with time for many such links to earlier work and within the new ideas to become established. This type of teaching distinguishes high attaining jurisdictions from those like England and the US ([See the work of] Askew and Hodgen)"*

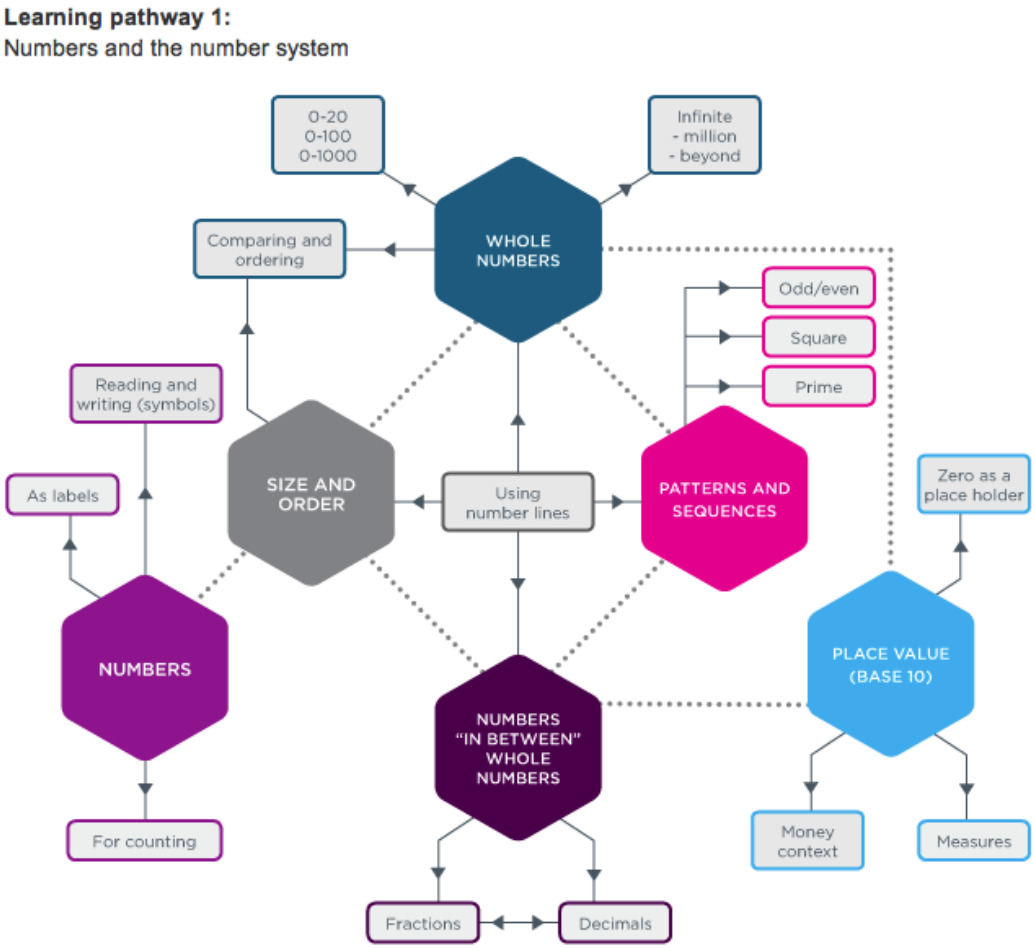
A respondent from a STEM organisation (writing in an individual capacity) emphasised the importance of links with other subjects: *"Little overt reference to this is contained in the tables. … There is, in particular, considerable potential benefit from looking at how better to interconnect learning in mathematics with that in science and in technology. Whilst I have not studied the suggested primary science curriculum I can guess that it will be very*

[16] Out of 47 respondents

conservative in addressing quantification. In fact mathematics is of critical importance in science and engineering, and overly descriptive approaches taken in schools are a source of great worry. When a skill is developed in maths, it is important for it to be exercised in science and technology. Such applications can be pointed to not only within maths teaching, but also within the core teaching of these other disciplines. The advantages are "win-win"; the pupils find their maths skills reinforced, and science learning itself is considerably deepened and enhanced. In some areas science and technology settings can actually lead maths input (e.g. in units of measure, order of magnitude, graphical representations, scale models ...)."

A small minority of respondents expressed a different view: *"I think the links are about curriculum in its wider sense and they do not need to be described in the PoS"* (Primary maths adviser); *"Creative teachers will find links to topic work and other subjects."* (Primary teacher)

The following diagram, showing links in primary school number, was recommended by participants in the July 3 workshop and others.



http://www.nationalnumeracy.org.uk/understanding-numbers-and-our-number-system/index.html

Question 5.3

5.3 Does the proposed mathematics curriculum promote the development of mental methods? Why / why not? If not, how could it be improved?

83%*[17] thought that the draft curriculum did not promote the development of mental methods and 8%[18] that it did. The main reason that respondents gave for their judgement that mental methods were not promoted was the strong emphasis given to formal written methods in the draft; this was thought to be regrettable as for practical purposes and that, for encouraging understanding, flexibility and agility, mental methods were seen to be more useful. It was pointed out that few, if any, people would use formal calculation methods in real life as devices with calculators are so readily available.

This response from a local authority respondent was typical of many. *"The curriculum does point to the development of mental methods. However, this could be clouded by earlier introduction of standard methods. There is a danger that an over-reliance on written methods of calculation developing with children becoming unwilling or unable to deploy mental strategies when these would be more appropriate. Clearer guidance and exemplification around mental methods would be helpful."*

One respondent commented, *"There is an inbuilt assumption* [in the draft] *that written methods are* [always] *more efficient and seem to be the goal even though choosing and using is set out in the aims. There are some contradictions between the expectations, especially in Y2:*
add and subtract numbers with up to two 2-digits including using column addition without carrying and column subtraction without borrowing [49]
[and]
add and subtract numbers mentally including:
- *a 2-digit number and ones*
- *a 2-digit number and tens*
- *two 2-digit numbers [50]*

Why do it as column addition/subtraction if you can do it mentally? The message to children is because this is the proper way of doing it and so they won't choose (we already see this is the biggest issue with children in tests; they have a range of methods but make really poor choices, often choosing a written method which is the not the best option for the numbers involved). The notes reinforce that the written method is the best. Introducing new elements of maths makes most sense when you are in a situation that forces you to do things differently and to think differently. Otherwise it just becomes a memory game with no reasons."

A primary teacher wrote of the proposed requirement for column methods: *"[M]y most concerning issue that I have is the inclusion of column method to be taught from KS2. I am a teacher with nearly 20 years experience in KS2. The introduction of horizontal methods and the development of mental maths*

[17] Out of 36 responses
[18] Out of 36 responses

strategies have in my experience proved to be an invaluable tool in developing pupils' understanding of place value. To remove a teacher's ability to utilise this method in favour of a method that can, for some pupils, inhibit their development of place value is doing a great disservice to them. I would be very interested to know what research this decision has been based on."

Question 5.4

5.4 Does the proposed mathematics curriculum enable all students to develop:

- procedures,
- conceptual understanding
- problem solving
- mathematical reasoning, and
- applications

so that all aspects of mathematical knowledge and skill will be taught to all students? If not, how and where could it be improved?

Generally, respondents said that the draft would enable pupils to develop procedures (73%*[19] 'yes') but not conceptual understanding, problem solving, mathematical reasoning and applications (100%[20] 'no' in each case).

Respondents to this question remarked again on some of the perceived difficulties mentioned earlier in this report: no mention of exploration and investigation; no mention of open-ended problems; no mention of generalising. If pupils were encouraged to work in these ways, it was reported by many, pupils would develop their powers of reasoning and explanation.

Many commented on the fact that problem solving seemed to consist of 'word problems'. A primary teacher said, *"I was surprised not to see a 'Using and Applying' section within the new curriculum, bearing in mind this is key to the application of all mathematics and stated as an aim. I think this diminishes the importance of 'rich' mathematical tasks and open-ended investigations, which are crucial to developing independent thinking, rather than children producing pages of formal calculations, then related word problems. The importance of this was highlighted in the recent OFSTED report, Mathematics: Made to Measure), stating that problem solving in schools is often limited to word problems. The new curriculum would seem to be promoting this narrow approach to problem solving."*

Question 5.5

5.5 Considering **number**:

[19] *Out of 22 respondents.

[20] Out of 34, 36, 28, 20 respondents respectively

- Will the proposed mathematics curriculum provide reliable knowledge and proficiency in arithmetic?
- Are the two routes of learning to number as described by ACME above (calculating and reasoning about quantities) adequately described and connected?
- Will the proposed mathematics curriculum provide an adequate basis and experience for applications of arithmetic, algebra and data interpretation?

Only 3%[21] of respondents replied positively to this question, although they did not disagree that there was *"plenty of calculating included"*. Few respondents commented specifically to the questions about ACME's two **routes of learning**, but several did mention that there was very little in the draft that would encourage reasoning about quantities.

A mathematics education research group did not see two separate routes: "We do not agree with ACME that calculating and reasoning about number are separate routes - effective calculation involves reasoning about quantities. These need to be taught in an interlinked way. As already noted there is not enough application work, nor enough focus on relationships between numbers which form the basis for algebra and data interpretation."

All comments emphasised the need for understanding, making links and application. A learned society representative wrote, *"The whole tenor of the draft is almost unbearably passive: pupils are 'taught'; they are expected to 'recognise' and to 'practise'. There is almost no mention of drawing, or weighing, or actively measuring, or making, or discovering, or exploring, or collecting, or describing, or comparing, or ... . This requires a shift of focus, but could be easily corrected by such changes as*

replacing "Pupils should be taught to:"
by "Pupils should:"

and by adjusting the remaining language accordingly".

Several respondents took the opportunity to comment in detail here on methods of calculation, particularly the suggested 'borrowing' method for subtraction, which, as mentioned elsewhere, was reported to be quite inappropriate.

A secondary mathematics teacher was among several who commented on the use of calculators: *"I welcome the statement that teachers need to consider how ICT can best be used to support the teaching of mathematics. I am however at variance on the non-use of calculators which can be used constructively and creatively to strengthen among other things a feeling for number and further concepts in mathematics were the pupil, while developing understanding in a new concept, for the moment does not need to be focussed more on the arithmetic. There would be a sad loss to the potential of their good use if the restriction suggested was imposed e.g. In [8] Year 1*

[21] Out of 29 respondents

Programme of Study: Key Stage 1 Notes and Guidance Ensure pupils practise counting in … recognition of patterns in the number system, could be enhanced through use of Calculator/ICT."

A respondent (role not given) said, *"This needs to be a 21st century curriculum that develops skills so that learners make appropriate use of technology. In the case of calculation this means developing skills of estimation and accurate use of a calculator. These skills, much needed in employment and further education, are developed by confident use of the calculator at an early age alongside mental and written strategies. Equating of the use of a calculator to a crutch for those who have difficulty calculating is poorly informed. The calculator is a respected device through which learners explore number facts and place value, thus developing their understanding of number. To leave the use of the calculator until the end of primary education is a missed opportunity to develop understanding of number."*

Question 5.6

5.6 For aspects of **number**, **geometry** and **measure** articulated in the proposed mathematics curriculum, consider their progression:

- Are topics introduced at the right stage, in the right way and in the right order? If you think not, say whether you are basing your opinion on organisational difficulty; inappropriate cognitive demands; inappropriate mathematical progression; etc.
- In particular, are the progressions for number, fractions and decimals suitably aligned with those for measures and other applications?
- Is the progression within each strand clear and how could the clarity of progression be improved?

Only nine respondents gave what could be classified as a yes or no answer to the question about stage and order of topics. None of these said 'yes'. There were respectively 21 and 10 'yes or no' answers to the next two parts; none of these said 'yes'.

There were many detailed comments about aspects of the programme of study with suggestions for improvement and clarification; there was clear dissatisfaction with aspects of progression. These comments will be worthy of further study when the draft is revised following consultation.

The small selection of quotations below gives a flavour of the responses.

- *"Measure and decimals do not relate to one another in a sensible way. Measure is a sensible way to introduce the concept of decimals and results in fewer misconceptions than money, but the programme of study does not introduce these ideas together."*
- *"Fractions objectives definitely too much too soon. 6 and 7 year olds have no experience in real life of thirds, so why learn it so soon? It's not something that is a regular part of their experience of*

measures - 2 litres of pop, a litre of fruit juice, a half litre bottle of water, a 250ml (1/4 l) carton of juice in their lunch, yes. Think about Liebeck's model of ELPS – (experience, language, picture symbolic) Thirds are meaningless to them." (Primary teacher)

- *"The use of standard measures before children have experienced non-standard measures and at such an early age is mad! The introduction of numerals in place value and then in fraction notation is going to be unnecessarily confusing for children in year 1 getting to grips with whole numbers. Teaching children to tell the time to the nearest 5 mins in Year 2 is unfair when conceptually they are not ready to understand this – believe me I have tried."* (Role not given)
- *"There are some discrepancies in the conceptual progression in learning time. (Some of which are ingrained and could be re-thought and corrected in this review.) SLOW DOWN.*
  There are two aspects of time:
  (i) Understanding what time is and what are the units for measuring it.
  (ii) Reading the clock.
  Using the instrument does not develop conceptual understanding. Reading the analogue clock requires reading of a different scale for each hand - really really complex and well beyond the stage of Y1 and Y2 pupils.
  Suggest we just talk about the passing of time in Y1 and leave 'telling the time' till later years.
  Y2 could talk about units for measuring time (hours, weeks, months, minutes) and pupils should know there are 60 mins in an hour, 7 days in a week etc.
  Why not make the link between analogue and digital from the start? e.g. Y1 [38] could be moved to Y3 and ask for telling the time as 7 o'clock AND 7.00; half past 7 and 7.30.
  Read the analogue clock in ONE direction (clockwise) only in the early stages. The 5 past 7 and 7.05 link up. Do quarter past but NOT quarter to. 35 mins past 7 and 7.35. i.e. Move and change [84] Linking ¼ to 7 and 6.45 is HARD. [131] Leave 'minutes to' until much later, say Y4.
  [182] Yes, 12 hour clock. But leave 24 hour clock until Y6." (Teacher education)
- *"… progression within shape seems poor – limited reference is made to irregular shapes and the levels of challenge/opportunities for reasoning are limited: making connections between properties of shapes and Van Hiele's levels of geometric thought could inform this further."* (Adviser)
- *"Progression within the geometry strand is unclear. Robots appear at [80] then disappear."* (Computer science researcher)
- *"Algebra and Probability do seem to have been dropped into Year 6 with no prior warning and Algebra (280-282) in particular does not appear to have any reason for being there. If it is called algebra and is limited to these few procedures, pupils will suddenly encounter a mysterious use of letters which may well be confusing and establish the attitude that algebra is best avoided in future. Much more*

guidance is needed as to why and how letters can be used to increase the efficient solving of some types of problems." (Primary teacher)

Question 5.7

5.7 If the proposed National Curriculum for mathematics were to focus on fewer things in more depth, what do you think should be prioritised and why?

The suggestion of focusing on fewer topics in greater depth was welcomed by 90%[22] of respondents. There was concern from all that pupils should understand the mathematics they learnt and be able to apply it confidently rather than to attempt to cover too much ground.

There was general agreement that number was the most important area of primary school mathematics, as indicated by a mathematics education research group: *"Clearly the main focus of the primary curriculum should be the number system, developing from whole numbers to rational numbers, and the number operations. Rational numbers derive from the need to measure continuous quantities, which means that measurement is a key area. However, to achieve depth in these, there need to be a variety of representations introduced and problems and questions both exploring mathematical structure and applying ideas to realistic situations. Some of these representations and applications will be geometrical and others data-related. Calculation should highlight mental fluency and ability to use technology. There is some trivial baggage around e.g. Roman numerals, written procedures, and binary numbers (which should be non-assessed and introduced for conceptual reasons but as was the case last time they were in the curriculum, teachers are more likely to focus on useless procedures rather than powerful ideas)."*

This from a primary teacher: "*Knowledge of number facts and number bonds, but also being able to apply them. Operations, again also in a practical sense as well as the method. Really solid understanding of place value; if this is really solid they will have no problem with measures and reading scales etc, even if this is taught separately much later."*

And from an infant teacher: *"Number and calculations first and foremost. Then measures and shape. Finally data. All areas should include problem solving and reasoning."*

A teacher education representative suggested the principle to be followed: *"Once a good set of aims is in place strip out any content that doesn't help to meet them… all the little things such as the bizarre km³ [301] should be omitted. The priority has to be that children end up thinking like*

[22] Out of 22 respondents

mathematicians… there's nothing in the proposed draft to suggest they will so far."

The focus on children thinking and behaving like mathematicians was also important to a professional association representative: *"There needs to be far greater emphasis on behaviours that enable children to become successful learners of mathematics, and rich experiences that help children to see the relevance of mathematics across the curriculum and to everyday life. Teachers need to focus on using and applying mathematics as well as developing fluency."*

And to a local authority: *"Priorities to ensure that children develop as mathematicians and have essential conceptual understanding to ensure best progress i.e.:*

- *Problem solving, reasoning and mathematical thinking and communication*
- *Properties of number and place value; fractions (but not including multiplication and division of fractions), decimals, percentages and proportion.*
- *Conceptual understanding of arithmetic; mental calculation; recall of fact; and written calculation (the latter in KS2)*
- *Application of number concepts through measuring, data and algebra*
- *Some basic work on geometry but with more focus on conceptual development rather than acquisition of skills and knowledge"*

An adviser suggested that algebraic ideas were important: *"Big ideas such as equivalence, additive reasoning, multiplicative reasoning, understanding the number system."*

Question 5.8

5.8 Does the proposed National Curriculum for mathematics make clear to teachers the relative importance to be attached to different areas of mathematics and topics within them? Should it? Why/why not?

89%[23] thought that the relative importance of the different areas was not made clear (or, perhaps, that they did not agree with the relative importance suggested).

There was a balance of views about whether the relative importance of different areas, in terms of recommendations of time to be spent on them, should be specified. A representative of a group of primary schools was in favour of specification: *"The relative importance to each area is not clear as there are no time scales given to any area for teachers to then map out the year. This could result in things being missed/not covered in enough depth and teachers running out of time." In contrast,* a local authority representative

[23] Out of 27 respondents

said, *"The current curriculum gives you suggested teaching times as a basis. However, I think that by not having the relative importance attached to the different areas of mathematics and topics within them would give teachers more opportunity to teach to the correct levels and 'plug the gaps' rather than feeling they have to teach topic x for a given number of days."*

A teacher in an all-through school pointed out the difficulties of achieving agreement about the relative importance of different areas of mathematics, saying that s/he considers estimation a more valuable skill than adding and subtracting fractions, but others may disagree.

A consultant said that defining the relative importance of different areas was the role of the non-statutory guidance.

6. Whole primary curriculum

Question 6.1

6.1 How much curriculum time do you think would be needed to implement the proposed mathematics curriculum in such a way as to achieve the aims eg for Key Stage 1? Key Stage 2? Number? Geometry and measures?

This question was interpreted in two different ways: some understood it to be asking how much teaching time would be needed, whereas a small number interpreted it as how much development time would be needed before the new curriculum was introduced.

As mentioned in the discussion of question 3.2, respondents reported that the amount of teaching time needed was at least as much as that allocated at present. Suggestions given for the mean daily amount of time (in hours) were: 1.5 (60 minutes on number and 30 on other areas); 1 – 1.5; at least 1, with as much use of cross curricular work as possible; 1 with opportunities to practise and apply skills and knowledge in other areas; more than 1; same as at the moment; 1.2 to 1.4; up to the school; more than at present; more than at present; 1.4; longer than we have now; daily maths lessons; 1 plus weekly mental maths and problem solving sessions and additional time to consolidate times tables and division knowledge to speed; 0.75; 1 at least but it depends on the school; substantially more than at present; 1 – 1.2; more than at present; *"literacy get more than maths"*; more time than at present; *"the simplest way of conveying the intended extent of the curriculum would be to follow most other jurisdictions by specifying a minimum number of hours to be devoted to mathematics lessons in each school year"*.

As to the time needed before a new curriculum could be introduced, both the responses said two years.

Question 6.2

6.2 Do you think this is an appropriate National Curriculum? Please explain your answer.

85%[24] stated that this was not an appropriate curriculum; many of the reasons for this judgement have already been mentioned. A major 'new' reason given was the perceived backward-looking nature of the proposals:

- *"No. It is so out of date! We need a curriculum for the 21st century not the 19th. Why is there no mention of ICT, the Internet etc."* (Freelance teacher)
- *"I think it could have been written 50 years ago – i.e. it makes no reference to modern technology. I feel there should be a recognition that children are surrounded by electronic media and should be learning how we control these technologies."* (Secondary teacher)
- *"No! I feel that it is setting primary mathematics teaching back 30 years. We are now beginning to teach in a more engaging way (see* …[work such as that of] *Nunes, Askew and Anghileri and research into the Netherland's Real Maths approach. Taking out using and applying will result in a dry subject, based on instrumental teaching and where children will be seen as failure for not remembering facts and procedures. It reduces maths to a memory exercise rather than relishing the challenge and insight that understanding brings."* (Primary teacher)
- *"No – not relevant for purpose or pupils in the 21st Century; for example, there is reference to "borrowing" as part of the language around subtraction – this term is confusing and meaningless and had been eradicated from classroom parlance in favour of more meaningful terms such as exchange / decomposition."* (University primary mathematics education lecturer)

One consultant unhappy with the proposal said, *"No. We will be the laughing stock of education experts in the rest of the developed world if we implement this curriculum."*

And a primary teacher pleaded: *"Where is the space for a proper and professional debate on these proposals?"*

7. For those familiar with progression into secondary school:

Question 7.1

7.1 To what extent does the proposed curriculum provide adequate preparation of knowledge, skills and capabilities for mathematics in secondary school and outside?

- in number?
- in geometry and measuring?

[24] Out of 40 respondents

- in other aspects of mathematics?

Are there any items in the proposed curriculum that you think do not provide an essential basis for further mathematical study? If yes, please list below and explain why.

Few respondents replied to this question and the next. Some said that, if pupils learnt all that was in the proposed curriculum, they would be well prepared for secondary school. Others said that, if pupils had been pushed to learn procedures that they did not understand, they would be short of confidence and not well prepared to move on with the secondary curriculum.

A researcher working with primary schools to teach mathematics using Cuisenaire rods stated, *"Student retention of mathematical learning is higher in the Cuisenaire-Gattegno approach, and this gives pupils a sounder base to build on in algebra, calculus and computer programming in secondary school."*

Another point made was that, as it is not yet clear what the Key Stage 3 curriculum would be, it was not possible to comment on how well prepared pupils would be.

A local authority representative wanted an all-through approach: *"I believe we should develop one curriculum that goes from 5 to 18 with clear steps that build a journey for learners through each concept."*

A representative of professional associations discussed in some detail all transitions from EYFS upwards. For the Key Stage 2/3 transition, s/he said, *"Secure foundations need to be laid for progression to secondary mathematics. Many topics in the draft Programme of Study are best left to secondary.*

For example:

- *speed – whilst this is an everyday phenomenon, formal treatment of this compound measure is hard for many secondary age students to understand;*
- *formal algebra – the patterns and structure of number and relationships between them are crucially important but early introduction of letters (often to represent objects rather than numbers) can lead to children never fully understanding algebra;*
- *the probability scale – this abstract concept is best left to secondary, but being able to understand likelihood in the context of games and assessment of risk is wholly appropriate at Key Stage 2;*
- *transformations – these are operations on shapes and should be defined precisely; this does not mean that children shouldn't make designs with shapes but it is best not to say this is a transformation when it is not being defined rigorously;*
- *scaling areas – working on similar shapes is challenging for many secondary learners; introducing this in primary is unhelpful.*

Ensuring that children have strong foundations in number so they can use associativity, commutativity, distributivity and equivalence to deal easily with calculations such as

- 12x75 = 3x4x25x3 = 3x3x100
- or 12÷75=(3×4)÷(25×3)=4÷25=0.16,
- or 3x17 + 7x17 = 10x17,

are essential for progression to algebra. This fluency and confidence with the inner structures within number problems cannot be achieved by routine practice of standard algorithms which do not encourage children to 'think before they do'."

Question 7.2

7.2 The ICCAMS project (http://tisme-scienceandmaths.org/the-tisme-research-projects/iccams/) highlights the lack of progress of our children in areas of multiplicative reasoning and algebra, relative to children 30 years ago.

- Does the draft curriculum ensure suitable foundations for children's later learning of ratio and proportion, and their application of multiplication and division in appropriate situations? If not, what changes would need to be made in order to achieve this?

- Similarly, does it ensure suitable foundations for all children to progress towards using algebra?

The question asked about current pupils' lack of understanding of multiplicative reasoning and algebra, and there was agreement amongst respondents that these were difficult areas but that the proposals would not solve the problem. It was reported that pupils needed more opportunity to explore situations practically and hence come to understand the connections between fractions, division, and ratio and proportion. It was pointed out that 'proportion' was mentioned only in the draft Notes and Guidance (and then only once) and not in the main programme of study except in the title 'Ratio and proportion'.

These quotes give some indication of the concerns:

- *"No – it appears to be only relevant for year 6. Children should be investigating rules and patterns in more than just number sequences to make generalizations, predications and support reasoning."* (Role not given)
- *"In as much as algebra can be conceived of as generalised arithmetic, this curriculum fails to provide a foundation. 'Carrying' and 'borrowing' procedures are not what is needed. Children need to understand the structure of number, including place value and relationships. Operations need to be thoroughly understood in the context of number before they can be applied in algebra. So for example, multiplication as both scale enlargement and repeated*

addition." (Consultant)

- *"Much greater focus on reasoning and solving problems through a range of methods would be required. More explicit focus on maths as a language and how that language develops and leads from knowns to unknowns would support progress towards algebra - rather than atomising content."* (University lecturer in mathematics education)
- *"The overly ambitious POS are likely to result in weaker foundations for algebraic and proportional reasoning. There is no emphasis on developing a secure understanding of the way number relations work including inverse relationships between operations that are crucial to later learning."* (Consultant)

More positively, a primary teacher wrote*, "I think it is positive that the curriculum attempts to introduce fractions into KS1 in greater depth and there is greater emphasis on counting in fractions; this will support proportionality. Missing number problems should continue through years 4 and 5 to ensure children are familiar with them in Y6. This will support algebra."*

8. And finally….

Question 8

What do you consider would be the main consequences – intended or otherwise – of the proposed changes to the primary mathematics National Curriculum and why? For any positive consequences, what other policy measures would better ensure that the benefits are realised? For any negative consequences, what changes would need to be made to reduce the risk?

This question asked what respondents thought would be the main consequences of the proposed curriculum. 87%[25] anticipated negative effects, for the reasons that have been mentioned at many points earlier in this report.

Some positive points were made:

- *"Positive: I do think the implied focus on number and sound knowledge of number facts is right, but as there is no indication of time allocation or weighting to each strand this may be lost."* (Primary teacher)
- *"Positive: Higher expectations. More control by schools."* (Infant teacher)
- *"Good focus on counting; fractions."* (Primary teacher)
- *"The move away from levelling is a positive; ICT being used (spreadsheets etc)"* (Web seminar participant)

[25] Out of 31 respondents

Several respondents suggested how things could be improved: *"The aims need to be looked at and celebrated and then make sure that within each yearly overview that the essence of the aims is repeated. Are the children mathematical thinkers?"* (Primary teacher)

"The changes needed are a) much more emphasis on meanings and applications, b) more realistic positioning of item against years b) removal of any specific methods d) embracing of modern technology in both the curriculum and teaching methods." (Mathematics education research group)

This respondent, whose rôle was not revealed, expressed some common concerns about copying the curriculum of other jurisdictions: *"I also know that many aspects of the draft curriculum have been influenced by the Ministry in Singapore and their primary mathematics curriculum. We need to be cautious of international comparisons with societies that bear little or no resemblance to ours and therefore their attitudes and needs for a mathematics curriculum will be very different to ours."*

As did this mathematics education research group: *"We already know that more than 50% of pupils in most high-performing jurisdictions find they need to attend coaching establishments in order to keep up with class work in mathematics. We should be clear whether it is the intention of this programme of study to force similar quantities of additional schooling in England. We also note that many low performing jurisdictions also have very ambitious curricula but most students fail to master these."*

A primary teacher expressed his/her concerns and those of others thus: *"Too much too soon in terms of expecting children to do things at a younger age could be catastrophic. I worry that more children will be left behind and hate maths, or have huge gaps in their knowledge. The OFSTED document Mathematics: Made to Measure makes so much sense, placing the value on U and A, not learning tricks and facts to pass tests. This new curriculum seems for focused on learning facts and formal methods, rather than understanding how maths works and how to learn."*

A web seminar participant said that it was important to focus on experiences that a child is entitled to have not what they are expected to be able to do. Another spoke of opportunities to learn rather than being taught.

Here is a concluding plea from a learned society representative: *"Finally, we hope that the Department will not forget the most important principles in any National Curriculum reform - namely to keep the profession on-side, to make the pace of change realistic, and to provide the necessary training opportunities to ensure effective implementation."*

And an offer from professional associations: *"As the expert primary group of the mathematics subject associations, we would like to offer our support to ACME during the unenviable task of compiling responses from across interested parties. We would also like it to be known that our members are happy to meet with Government representatives, read and respond to any*

proposals, and offer suggestions for how the ATM and the MA could support the DfE to develop and implement ideas."

Anne Haworth
July 2012

Appendix: List of respondents

Organisations

Association of Teachers of Mathematics (ATM) and Mathematical Association (MA) Joint Primary Expert group
Coddington Primary School
Education Publishers Council - Mathematics Publishers
Education Gateshead
Essex Consultancy Team
London Mathematical Society (LMS)
Mathematics and Science Education Research Centre, University of Southampton
Mathematics Education Research Group, Department of Educational Studies King's College London
Mathematics in Education and Industry (MEI)
Prees CE Primary School and Nursery
Royal Statistical Society (RSS)
Scholars Consulting
Service Children's Education
Sociality Mathematics CIC
Staffordshire LA
Tameside Mathematics Specialist Teachers Cohort 2
The Brook School

Individuals

Adrienne	Amos	Teaching (primary)
Simon	Ansell	Teaching (primary)
Jayne	Applegarth	Teaching (primary)
Rachel	Atkinson	
Helen	Bellamy	Teaching (primary)
Richard	Boyce	Teaching (primary)
Sarah	Brammer	Teaching (primary)
Siree	Brown	
Stefanie	Burke	
Hugh	Burkhardt	Higher Education
Jayne	Campling	
Helen	Clarkson	Local Authority
Jinny	Crossley-Klinck	Local Authority
James	Davenport	Higher Education
Penny	David	Teaching (primary)
Caroline	Dawes	Local Authority
Marie	Dixon	Teaching (primary)
John	Dore	Higher Education
Ruth	Esplin	Teaching (primary)
Andy	Fenton	Teaching (primary)
Nicola	Flisher	Teaching (primary)
Vicki	Giffard	Independent Consultant
David	Green	Teaching (primary)
Katie	Green	Teaching (primary)
Helen	Hackett	Teaching (primary)
Jane	Haddock	Independent Consultant
Pete	Hall	Consultant

Cathryn	Hardy	Independent Consultant
Rachel	Harrod	
Trisha	Henley	Teaching (primary)
Darren	Hilton	Teaching (primary)
Rosemary	Hobbs	Teacher (secondary)
Kirsty	Holden	
Helen	Humble	Teacher (secondary)
Ian	Jones	Education Research
Joseph	Leppington	Teaching (primary)
Martin	Little	Education Research
Charlotte	Madine	Independent Primary Consultant
Katie	Mallinson	Teaching (primary)
David	Martin	
Toni	McPherson	Primary Teacher Training
Debbie	Morgan	MaST Programme
Zara	Murton	Teaching (primary)
Adella	Osborne	Primary Teacher Education
Pauline	Palmer	Primary Teacher Education
Rachel	Peckover	Teaching (primary)
Sue	Pope	Mathematics Education
Caroline	Rickard	Primary Initial Teacher Training
Alan	Roach	Higher Education
Rosemary	Russell	Mathematics Adviser
Hannah	Sheard	Teaching (primary)
Sue	Smith	Teaching (primary)
Jude	Stratton	Mathematics Education
Robert	Tait	Teaching (primary)
Michael	Tidd	
Victoria	Tilley	Local Authority
Ruth	Trundley	Primary Mathematics Adviser
Helen	Williams	Teaching (primary)
Tony	Wing	Primary Mathematics Consultant

There are 84 respondents listed above; a further 14 respondents asked for their responses to be confidential and are therefore not listed. A small number of additional responses were received after this summary was created and have not been included in the analysis.